orientation to that in the newer Japanese city plan maps. Checking the feature in relation to other nearby features on both maps provides some additional assurance of a good fit. The aerial photographs were extremely valuable in this regard. As noted above, they were georeferenced to the exact same set of control points.

In some cases, such as the Postal Savings Building (Chokin Kyoku Building) in Hiroshima, the decision was made to not use the feature because the depiction on the Army map was far too incorrect. The depiction on the Army map was either too small or there was no assurance as to what part of the building outline on the new city map it would correspond. Curiously, the outline on the wartime aerial photograph was very similar to that on the newer Japanese city plan maps, and not to that on the U.S. Army map. In other cases, the feature is used with some judgment. Bridges, for instance, are often somewhat incorrect in outline on the Army map, and the GIS user can make some additional judgment by examining riverbanks and associated roadways on the aerial photograph and the newer Japanese city plan maps. Having both available, along with additional information from other sources, helps to establish that river banks near the bridges used as control points for this work were not moved in the interim period 1945-1979 due to dredging and filling, land reclamation, construction of riverside parks, etc. This allows a more accurate identification of the river channel at either margin or the midpoint.

Results. The results of the map alignment process are summarized in terms of the fitted coefficients of the first-order affine transformation for each of the map image rasters in Table 9. Since the newer Japanese city plan maps consist of multiple contiguous tiles of identical dimensions, a representative tile is shown for each city. The coefficients A and D are the decimal degrees longitude and latitude in the geographic coordinate system based on the Tokyo datum for the topmost, leftmost pixel in the map image raster. The scale of each image is determined primarily by the coefficients B and F. It should be noted that the coefficients F will typically be about the same for both cities for a map of the same scale, but the coefficients B will differ slightly due to the different numbers of meters per degree of longitude at the latitudes of the two cities, respectively. The coefficients C and E are smaller and represent primarily a rotation between the two map systems.

Table 9. Georeferencing values for map image rasters

	Hiroshima		Nagasaki	
	U.S. Army map	Japanese city plan map[a]	U.S. Army map	Japanese city plan map[b]
Raster size, pixels				
X	15,974	18,400	11,200	16,340
Y	14,400	14,400	14,374	11,725
Transformation coefficients				
A, degrees	132.3925806	132.4302620	129.8269045	129.8606711
B, degrees/pixel	$\mathbf{8.760077202 \times 10^{-6}}$	$1.725893206 \times 10^{-6}$	$8.462911195 \times 10^{-6}$	$1.69262436 \times 10^{-6}$
C, degrees/pixel	$5.495370847 \times 10^{-8}$	$-3.236563939 \times 10^{-9}$	$8.271791250 \times 10^{-8}$	$-3.81910236 \times 10^{-9}$
D, degrees	34.4271408	34.39737835	32.79729086	32.7845609
E, degrees/pixel	$-3.648336953 \times 10^{-8}$	$-3.9907473963 \times 10^{-9}$	$1.917003647 \times 10^{-7}$	$1.91052351 \times 10^{-9}$
F, degrees/pixel[c]	$-7.183456916 \times 10^{-6}$	$-1.431263862 \times 10^{-6}$	$-7.180363949 \times 10^{-6}$	$-1.42575517 \times 10^{-6}$

[a]Tile O-8 (Each Hiroshima tile is 2.5 km × 1.75 km.)
[b]Tile 51-88-2 (Each Nagasaki tile is 2 km × 1.5 km.)
[c]Pixel numbering is from top to bottom in the raster, whereas degrees' latitude increases from the bottom of the raster to the top.

Hiroshima. The U.S. Army map was fitted with a resulting mean absolute difference of about 9 m for the 23 control points used. The largest errors were in control points furthest from the centroid of the control point aggregate, as one would expect. The aerial photograph was fitted with a considerably smaller mean absolute difference of only about 5 m, on the same 23 control points. The discrepancies obtained for all of the points are shown in Table 8 and are depicted in three-dimensional plots in Figure 10.

It is apparent in the plot that the discrepancies at the Yokogawa Bridge and the Kodo National School stand out as being larger than those at the other control points. These two control points were therefore checked in various ways and the implications of their discrepancies were considered in various respects:

1) The effect of these two points on the location of the hypocenter on the newer city plan map was carefully checked and found to be small (not more than about one meter).
2) Their fractional effect on the estimated parameters is small except for the cross-term parameters that define the rotation and skewing of the map, and the latter are small terms; therefore their effect on map alignment is small and mainly affects areas near them, i.e., in the west and northwest directions from the hypocenter. Key examples: the difference in the fitted location of a point near Yokogawa Bridge, north-northwest from the hypocenter at about 1.3 km, is about 5 to 6 m, and the difference in the fitted value for a point near Hijiyama Bridge, east-southeast from the hypocenter at about 1.7 km, is about 2 to 3 m.
3) Because these two points have such a small effect on the main estimated scale parameter B that relates the X coordinate of pixel address to longitude, they cannot be responsible for the observed departure from the map's nominal scale factor (the "1.3% stretching") that was observed in the fitted U.S. Army map.
4) When these two points are examined with respect to their discrepancies in the fitting of the war-era aerial photograph, it is seen that the discrepancy in both cases is much smaller than the discrepancy for the U.S. Army map. This is consistent with the more general result for the aggregate of 23 control points: that the inaccuracy is in the U.S. Army map and not in the newer city plan maps.
5) Another important question is whether the misplacement of these two features on the U.S. Army map is representative of the misplacement of other nearby features. To the extent that this is true, the inclusion of these features improves the fit for all features close to them, at an expense of degrading the fit very slightly for features in other areas. It does appear to be the case for the area near Yokogawa Bridge that the neighboring area is misaligned similarly to the bridge. It does not as unequivocally appear to be the case in the area near the Kodo National School.
6) In a more general sense, it is notable that both of these points are in the west and northwest directions from the hypocenter. This aspect is of more concern because there are few control points in that direction. A related factor is that this area underwent unusually extensive reconstruction after the war, including the rerouting of entire river channels subsequent to a large flood: the Yamate-gawa and Fukushima-gawa rivers were combined into a new, large channel called the Hosuiro (drainage canal). Examination of the maps and aerial photographs does suggest that the U.S. Army map as aligned for this work is generally more discrepant at locations more than about 1.2 km north of the hypocenter than at locations less than 1.2 km north of the hypocenter, although the alignment is fairly good at distances further south and about as far west as the westernmost control points; i.e, to at least 1.2 to 1.5 km west. This is

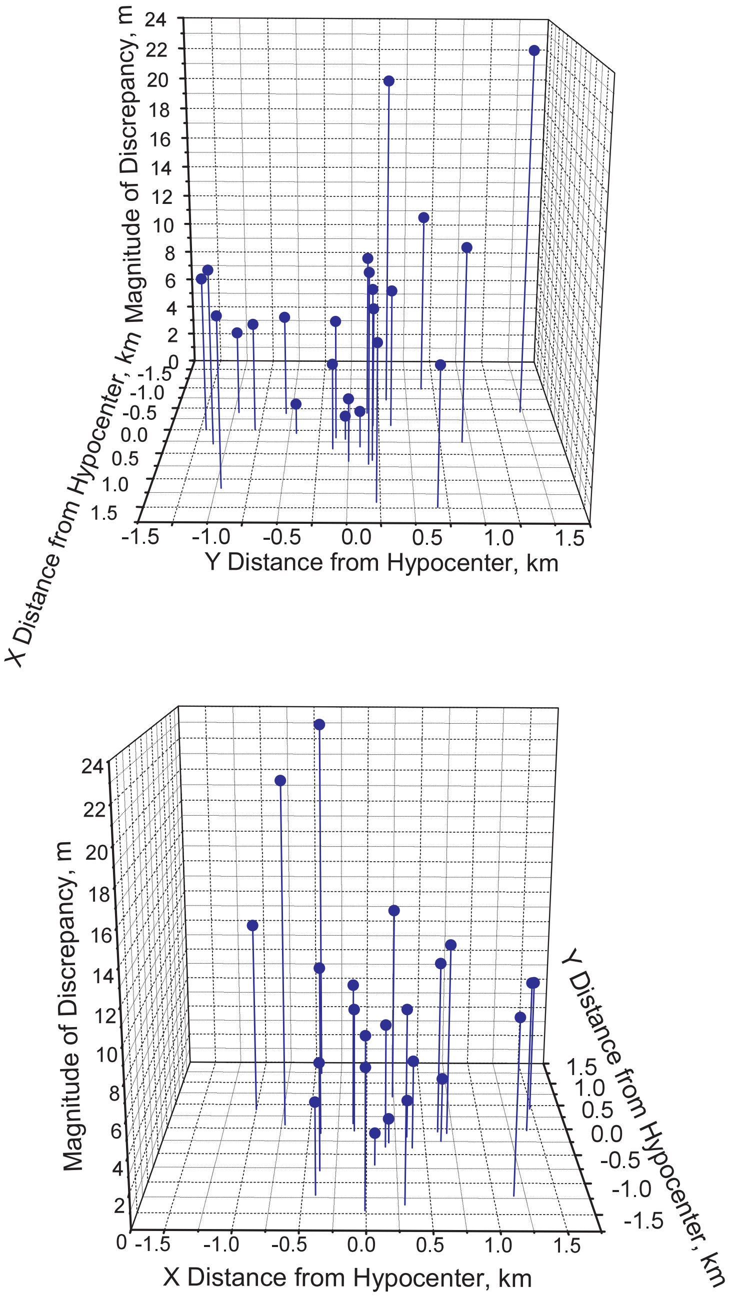

__Figure 10.__ Residual discrepancies remaining in the locations of the 23 control points on the fitted U.S. Army map (distances from locations on newer city plan map to corresponding locations on U.S. Army map), two views.

at least partially due to the distances involved; i.e., it is consistent with the overall trend that discrepancies are greater at longer distances from the hypocenter. This issue is discussed further below in this section.

7) On balance, it was decided to retain these two control points in the georeferencing.

The mean absolute difference of about 9 m obtained in this work compares quite favorably with the figure of 22 m given by Hubbell in his 1986 report (section entitled “The Work of Hubbell” above), but this observation is qualified by the fact that only a few of the control points used here are the same as those used by Hubbell. The transformations determined by Hubbell by his use of least-squares minimization of the discrepancies for his 18 control points, and separately by visual inspection of the superposed maps, equation (3) in the section entitled “The Work of Hubbell” above, were coded into a spreadsheet and compared with the transformation recommended in this work, on the set of 23 map locations used as control points for this work. The results are given in detail in Table 10 for the transformation of this work and Table 11 for the transformation that Hubbell determined by visual inspection. Both transformations defined by Hubbell have a mean absolute error of about 13.5 m for these 23 locations, as opposed to about 9.0 m for this work. The transformed values given by this work are centered, on average, within a few tenths of a meter on the depicted locations on the newer city plan map, whereas those given by Hubbell are on average about 5.5 m east and 5.5 m south of the depicted locations (method based on visual inspection) or about 2.5 m east and 6.5 m south (method based on least squares for Hubbell’s 18 points).

If one assumes that the nominal scale and rotation values of the maps as used by Hubbell apply to the work by Fukuken Choosa Sekkei and RERF staff, discussed in the section “The Work of Fukuken Choosa Sekkei” above, a transformation similar to that of Hubbell is defined. It differs only by about 2 m in the offset, as reflected by the difference in hypocenters; i.e., the transformed coordinates for the 23 control points would be on average about 7.5 m east and 6.0 m south of the depicted locations on the newer city plan map. The results of the map alignment with respect to the individual alignment of other key landmarks near the hypocenter, apart from those represented in the 23 control points, are discussed in below in the section on “Implications for the Locations of the Hypocenters on the Newer Maps.”

An interesting aspect at Hiroshima is that a stretching of about 1.3% in the east-west direction produced the best fit (the “B” coefficient shown in boldface text in Table 9). This was checked to be sure that it is not a result of one or two aberrant control points. The resulting fitted map has excellent agreement in the east-west direction with regard to a number of major streets and river channels. The source of this small distortion in the U.S. Army map is not clear, but this study provides strong empirical evidence that the alignment with this stretching produces the most accurate relationship between the two maps.

A close visual examination of the map alignment developed in this work confirms that many streets and river channels are well aligned. Many streets in the central region of Hiroshima within 1-2 km from the hypocenter are very well aligned, including streets running nearly east-west as well as those running nearly north-south, although streets west of the Otagawa river are difficult to evaluate because the postwar reconstruction in that area is so extensive. One notable exception is the small street running north-northeast from an intersection near the east end of the Motoyasu bridge, which ran along the west side of the Shima Hospital building; hence, very close to the hypocenter. This street is located about 13 m too far to the west on the U.S. Army map, a fact that is confirmed by the aerial photographs, and its misplacement appears to have been a key factor in the earlier map alignments.

In addition, the banks of the Otagawa and Motoyasugawa rivers in areas within about 1 km of the hypocenter are well aligned with two notable exceptions:

1) the east bank of the Otagawa along the west side of Nakashima island (at the north end of which is now Peace Park), and
2) the east bank of the Motoyasugawa and Otagawa north of the Motoyasu Bridge.

The most important of these relationships can be seen in Figures 11 and 12. Additional details about the alignment of landmarks near the hypocenter, and the effect of these landmarks on the earlier alignments, is given in the section "Implications for the Locations of the Hypocenters on the Newer Maps" below.

Table 10. Details of discrepancies for the transformation of map coordinates defined by the map alignment recommended in this work, 23 control points in Hiroshima

Location	Army map coordinates, kyards		Transformed coordinates on Japanese city plan map (km)		Discrepancy vs. depicted location on Japanese city plan map (m)		
	X	Y	X[a]	Y[a]	X[a]	Y[a]	SQRT (X^2 + Y^2)
Motoyasu Bridge, mid-river	744.176	1261.652	26.611	-178.448	-6.8	3.5	7.7
Center of dome of A-bomb Dome	744.173	1261.825	26.602	-178.291	-5.1	7.2	8.8
Fukoku Seimei Bldg, SW corner	744.488	1261.4	26.907	-178.670	-0.6	-1.4	1.5
Teikoku Bank, NE corner (Andersen's)	744.669	1261.521	27.071	-178.556	0.2	-2.3	2.3
Bank of Japan Hiroshima Branch, SW corner	744.456	1261.308	26.882	-178.756	-4.7	-5.8	7.5
Honkawa Bridge, mid-river	743.862	1261.611	26.321	-178.493	-10.1	-1.8	10.2
Naka Telephone Bldg, SW corner	744.698	1261.278	27.107	-178.777	-5.1	-1.6	5.4
San'in Godo Bank, NW corner	744.961	1261.622	27.338	-178.456	-5.7	-8.8	10.5
Kirin Beer Hall, NE corner	744.983	1261.41	27.366	-178.649	-3.2	-2.3	3.9
Chuden Bldg, SW corner	744.349	1260.95	26.795	-179.086	1.7	1.0	1.9
Fukuya Dept. Store, NW corner	745.045	1261.59	27.417	-178.484	-7.1	-9.3	11.7
Hiroshima Castle moat, SW outside corner	744.589	1262.44	26.965	-177.718	12.1	1.9	12.2
Kodo National School, center	743.525	1261.805	26.002	-178.325	15.8	-13.8	21.0
Yorozuyo Bridge, mid-river	743.851	1260.845	26.338	-179.194	2.9	-5.7	6.5
City Hall, SW corner	744.243	1260.565	26.710	-179.440	-1.3	6.8	6.9
Hiroshima Telephone Co. West Branch SW corner	743.234	1262.166	25.722	-178.002	11.8	-1.6	11.9
Meiji Bridge, mid-river	743.811	1260.393	26.316	-179.608	-5.3	-1.0	5.4
Yokogawa Bridge, mid-river	743.873	1263.064	26.282	-177.166	-15.7	17.0	23.1
Kyo Bridge, mid-river	745.807	1261.63	28.120	-178.428	6.4	6.7	9.3
Red Cross Hospital, NW corner	744.230	1260.122	26.712	-179.845	8.4	5.0	9.7
University "E" Bldg, NW corner	744.569	1260.213	27.023	-179.754	-10.6	2.8	10.9
Sakae Bridge, mid-river	745.908	1262.118	28.197	-177.980	5.1	-6.5	8.3
Hijiyama Bridge, W end at river bank	745.497	1260.354	27.878	-179.602	8.0	6.2	10.1
Mean:					**-0.4**	**-0.2**	**9.0**

[a]In the Japan land survey system, X is used as the north-south coordinate and Y as the east-west coordinate. The opposite is used here, for simplicity of comparison to the U.S. Army map coordinates.

Table 11. Details of discrepancies for the transformation of map coordinates defined by the map alignment determined by Hubbell based on visual inspection of superposed maps, 23 control points in Hiroshima

Location	Army map coordinates, kyards		Transformed coordinates on Japanese city plan map (km)		Discrepancy vs. depicted location on Japanese city plan map (m)		
	X	Y	X[a]	Y[a]	X[a]	Y[a]	SQRT (X^2 + Y^2)
Motoyasu Bridge, mid-river	744.176	1261.652	26.625	-178.452	7.1	-0.3	7.1
Center of dome of A-bomb Dome	744.173	1261.825	26.616	-178.296	9.6	1.8	9.8
Fukoku Seimei Bldg, SW corner	744.488	1261.4	26.914	-178.676	5.8	-6.7	8.8
Teikoku Bank, NE corner (Andersen's)	744.669	1261.521	27.075	-178.561	4.2	-7.4	8.5
Bank of Japan Hiroshima Branch, SW corner	744.456	1261.308	26.888	-178.760	1.3	-9.7	9.8
Honkawa Bridge, mid-river	743.862	1261.611	26.337	-178.498	5.9	-6.4	8.7
Naka Telephone Bldg, SW corner	744.698	1261.278	27.107	-178.783	-4.6	-7.6	8.9
San'in Godo Bank, NW corner	744.961	1261.622	27.340	-178.462	-3.3	-14.6	15.0
Kirin Beer Hall, NE corner	744.983	1261.41	27.366	-178.654	-2.6	-7.3	7.7
Chuden Bldg, SW corner	744.349	1260.95	26.797	-179.091	4.4	-3.9	5.9
Fukuya Dept. Store, NW corner	745.045	1261.59	27.421	-178.493	-3.0	-19.0	19.2
Hiroshima Castle moat, SW outside corner	744.589	1262.44	26.982	-177.723	28.5	-3.6	28.7
Kodo National School, center	743.525	1261.805	26.022	-178.328	36.0	-16.6	39.6
Yorozuyo Bridge, mid-river	743.851	1260.845	26.344	-179.199	9.6	-10.2	14.0
City Hall, SW corner	744.243	1260.565	26.709	-179.445	-2.4	1.6	2.9
Hiroshima Telephone Co. West Branch SW corner	743.234	1262.166	25.750	-178.005	40.1	-4.7	40.4
Meiji Bridge, mid-river	743.811	1260.393	26.318	-179.612	-3.3	-5.2	6.1
Yokogawa Bridge, mid-river	743.873	1263.064	26.313	-177.170	16.1	13.0	20.7
Kyo Bridge, mid-river	745.807	1261.63	28.111	-178.435	-2.6	-0.3	2.7
Red Cross Hospital, NW corner	744.230	1260.122	26.707	-179.850	3.1	0.1	3.1
University "E" Bldg, NW corner	744.569	1260.213	27.015	-179.759	-18.7	-2.4	18.9
Sakae Bridge, mid-river	745.908	1262.118	28.195	-177.988	3.6	-14.2	14.6
Hijiyama Bridge, W end at river bank	745.497	1260.354	27.861	-179.611	-8.9	-2.5	9.3
Mean:					**5.5**	**-5.5**	**13.5**

[a]In the Japan land survey system, X is used as the north-south coordinate and Y as the east-west coordinate. The opposite is used here, for simplicity of comparison to the U.S. Army map coordinates.

The four more distant points used by Hubbell in his set of 18 points, at distances of about 3.1 to 4.2 km, were checked under the DS02 alignment and yielded a mean absolute difference about the same as Hubbell's transformation: 48 m vs. 46 m, with a range from 38 m to 58 m. Most of the discrepancy for these particular points is in the north-south direction, for both the alignment of this work and that of Hubbell. A large portion of the 22-m mean absolute difference reported by Hubbell for his 18 points appears to have been due to these 4 most distant points. The cause of the larger discrepancies at these more distant points is not clear at present. Errors due to intrinsic limits of the plane rectangular coordinate system become larger over such distances, but not

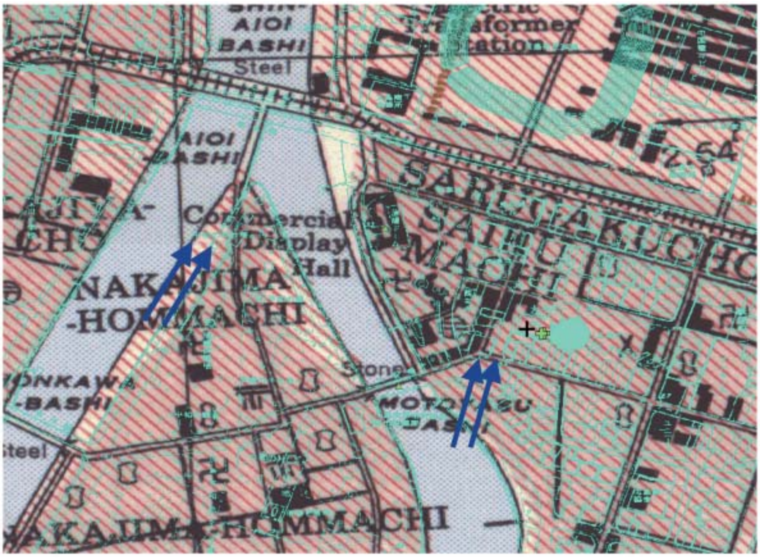

Figure 11. *The area near the hypocenter in Hiroshima. The new Japanese city plan map is overlaid in green on top of the U.S. Army map. The black cross is the new suggested location for the hypocenter on the new Japanese city plan map and the green crosses are the previous estimates. The blue arrows indicate differences in placement of two key features that affected map alignment.*

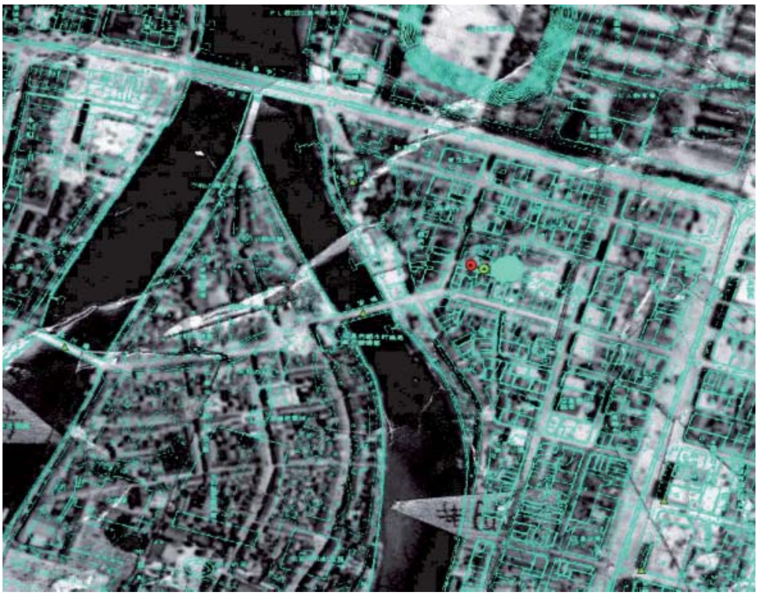

Figure 12. *The area near the hypocenter in Hiroshima. The new Japanese city plan map is overlaid in green on top of the pre-bombing aerial photograph.*

nearly enough to explain the discrepancies. Errors also increase with distance from the centroid of the control points, because there are no control points at these distances. Unfortunately, it was not possible to evaluate the effect of including these points on the alignment used in this work. Three of the four locations lie in map tiles of the newer city plan map that are not within the range of scanned map images that were available for this work. Furthermore, it was not feasible for this project to perform careful re-measurements and historical research on these locations to determine their true suitability for map alignment. There remains an open question as to whether the alignment determined for this work could be improved at distances >2 km, where "distal" survivors are located, by finding and properly documenting additional control points at such distances. However, it may also be observed that improving the fit of the U.S. Army map at such distances may not be necessary, because samples have not been accurately measured at such distances, and the improvement of survivor location data depends on the use of war-era aerial photographs to locate survivor shielding history neighborhood diagrams in relation to the newer city plan maps, and not on improving the fit of the U.S. Army maps.

Nagasaki. The U.S. Army map was fitted with a resulting mean absolute difference of about 7.5 m for the 9 control points used. The values obtained for all of the points are shown in Table 12. The stretching observed in Hiroshima was not produced by the Nagasaki fitting: the best fit had a stretching of about 0.15% on the east-west axis. This value is small in light of the range of the errors inherent in the resolution of the printing on the original paper maps and similar sources of error. It should also be observed that the relatively small number of control points available in Nagasaki, as well as their very narrow extent along the east-west axis, provides substantially less information about the effective scaling in the placement of features on this axis than is the case for Hiroshima.

Implications for the Locations of the Hypocenters on the Newer Maps

It is presently not feasible to reevaluate in detail all of the work on the hypocenters performed by Hubbell et al. (1969) and Kerr and Solomon (1976) in the context of the newer Japanese city plan maps, as discussed under "Georeferencing" above. The only way to define the locations of the hypocenters on the newer Japanese city plan maps is to use the U.S. Army map coordinates

Table 12. Nagasaki control points and final discrepancies vs. new Japanese city plan map: U.S. Army map

Control Point	GR (m)	Difference (m)
Yana Bridge, midriver	305.0	4.01
Chimzei Academy Middle School NE corner	449.5	0.87
Corner of stone wall at SW corner of Urakami Cathedral grounds	453.5	14.13
Shiroyama Elementary School NE corner	486.9	13.33
Motoki Bridge, midriver	747.6	6.04
Nagasaki Commercial High School	1107.5	8.39
Fuchi Middle School SE corner	1198.4	8.35
Mitsubishi Shipbuilding Laboratory, center of long bldg	1641.7	6.17
Japan Refrigeration Co. Inasa Ice Plant, center of bldg	2058.4	6.51
Mean absolute discrepancy:		**7.53**

given by Hubbell et al. and Kerr and Solomon for the hypocenters and find the corresponding points on the newer Japanese city plan maps that are defined by the map alignment performed for this work. As Hubbell took pains to point out in his 1986 report, because the map alignment is an affine transformation, no different result would be obtained by transforming all of the coordinates of locations on which the original triangulations summarized in ABCC TR 3-69 (Hubbell, Jr. et al. 1969) and Kerr and Solomon (1976) were based (locations of shadows, etc.), and re-doing the triangulations in the context of the newer city plan maps. Some differences might be obtained, of course, if the locations where shadows were measured could be identified directly from features depicted on the newer city plan maps, rather than by transforming their coordinates from the U.S. Army maps. This would be difficult if not impossible for many of the locations, because the structures involved were moved or demolished shortly after the war, and such an effort is clearly beyond the scope of the present work. Therefore, this work relies on the method of using the map alignment to transform the hypocenter locations that were determined in the earlier works just noted. The geometrical implications of this method are discussed in more detail in the section on "Geometrical Considerations Relating to the Position of the Hypocenter" below.

Hiroshima

The main focus of the original work on aligning the maps was to determine a location of the hypocenters on the newer Japanese city plan maps that would correspond to the locations that were defined in the past. The extensive work done in the past was carried out completely in the context of the U.S. Army maps, often transferred from older Japanese maps (Hubbell et al. 1969) and resulted in a Hiroshima hypocenter location at U.S. Army map coordinates of (744.298, 1261.707). The fitting discussed above suggests a location for Hiroshima for the new alignment produced by the georeferencing of the U.S. Army map to the new city plan map: it is 132.457273° longitude and 34.391361° latitude in the system based on the Tokyo datum and the Bessel ellipse, or (26.721, -178.395) in the kilometer coordinates of the map grid system of the Japanese city plan maps. This is about 15 m west-northwest from the previously determined location defined by the work of the Fukuken Choosa Sekkei company and by RERF staff, about 13 m west-northwest of the location suggested by Hubbell based on his repetition of the process used by Fukuken Choosa Sekkei company, and about 11 m northwest of the location suggested by Hubbell's numerical optimization of 18 presumably unmoved objects (see the above sections on "The Work of Fukuken Choosa Sekkei" and "The Work of Hubbell," respectively).

In order to assess the effect on the hypocenter location on the new city plan map that is caused by the 1.3% stretching of the U.S. Army map on the X-axis, a fitting was done that was constrained in such a manner as to allow only translation of the Army map image. This mimics the original work that was done by using the plastic overlay, but produces a result based on a numerical optimization of the 23 control points chosen for this work. The Hiroshima hypocenter location so determined was at (26.723, -178.395), 2 m due east of the location determined by optimizing the full, six-parameter linear affine transformation. Thus, about 2 m of the 11 to 15 m difference between the recommended hypocenter location of this work and those of previous work can be ascribed to the stretching. This location given by the constrained fitting is about 9 m northwest of the location that Hubbell determined by using his 18 points, which is the earlier result to which it is analogous and most directly comparable.

Figures 11 and 12 display two very important images of map overlays in the vicinity of the hypocenter. In the first figure, the U.S. Army map, georeferenced by the 23 control points as described above, is shown with the Japanese city plan map superimposed. In the second figure, the larger of the two wartime aerial photographs, also georeferenced by the 23 control points as described above, is shown with the Japanese city plan map superimposed. The fits of both the Army map and the photograph are quite good in the streets to the east of the hypocenter. The Army map shows a lack of fit at the Aioi Bridge, Motoyasu Bridge, and A-bomb Dome, whereas the fit of the aerial photograph at those locations, two of which were control points for the fitting, is much better. Comparison of the U.S. Army map and the aerial photograph to each other suggests that the locations of these features on the Army map are slightly misplaced. Similar observations about these same features were made by Hubbell and described in his 1986 report.

Even more importantly, a similar series of comparisons suggests that the Shima Hospital and the street along the front side of Shima Hospital, i.e., just west of Shima Hospital, are also slightly misplaced on the U.S. Army map, as noted above in the section Results (Hiroshima) under “A Method of Map Alignment Using the GIS.” This observation was not made in the earlier work (“The Work of Fukuken Choosa Sekkei” and “The Work of Hubbell” above). When the maps are aligned in the positions suggested by that earlier work, the street along the west side of Shima Hospital, and the riverbank areas noted above as being somewhat misaligned in the present work, are much better aligned. Prior to having the aerial photograph and both maps available to make such direct comparisons in the form of georeferenced, superimposed images, the exact nature and the full implications of these small discrepancies, on the order of 10-20 m distance, were not clear. Hubbell’s attempt at least-squares optimization was a step in the right direction, quite literally, in that it moves the hypocenter on the newer city plan maps several meters to the west from the locations determined by visual inspection. However, it was not independent of the alignments based on visual inspections, which were influenced by the misplacement of the features just mentioned.

Status of Key Landmarks Near the Hypocenter

It is of interest to note the following information about key landmarks, based on information from the book *Architectural Witnesses to the Atomic Bombing: A Record for the Future* (Hiroshima Memorial Peace Museum 1996), along with the comparisons of the maps to pre-bombing aerial photographs:

1) The A-bomb Dome remains in its original location as a ruin, preserved as an historical site.
2) The Shima Hospital building was destroyed in the bombing and was rebuilt on the same lot in a different shape and position. The streets around it appear from the aerial photograph to have essentially the same positions today as at the time of the bombing.
3) The building at the west end of the Motoyasu Bridge, which was originally a kimono shop and then became a fuel authority administration building during the war (Nenryo Kaikan), remains in its original location and now serves as a rest station for visitors to Peace Park.
4) The Motoyasu Bridge sustained little real damage and was repaired and used until 1992, when it was rebuilt in the same location.
5) The Aioi Bridge (“T-bridge”) was not destroyed in the bombing nor was it moved before 1979 when the new Japanese city plan map discussed in this work was issued. It was repaired and used until 1983, when it was rebuilt.

6) The streetcar tracks to the east of Aioi Bridge were moved slightly north during the reconstruction after the war, which straightened the approach of the street car to the bridge.

Geometrical Considerations Relating to the Position of the Hypocenter

It is important to consider the effect of the stretching that is implicit in the georeferenced image of the U.S. Army map, as to whether it affects the position of the hypocenter incorrectly. It seems clear from simple geometric intuition that the relative positions of features on a map image do not change if it is uniformly magnified, translated, or rotated, but this may not be intuitively clear in the case of the one-dimensional stretching discussed here. The fact that the transformation is affine assures that stretching preserves the relative positions of the objects on the map. Stretching the map in the X direction of course preserves the relative X locations of all features with respect to each other. If feature “a” is west of feature “b,” it does not become east of feature b because the map is stretched. More importantly, if feature “a” is twice as far from feature “c” as from feature “b” in terms of X coordinates, stretching the map uniformly on the X-axis does not change this ratio.

It is a fundamental matter of geometry that stretching the U.S. Army map does not change the location of the hypocenter in relation to any of the features on the U.S. Army map in the geometrical sense that is important to this work. Thus, the best approximation of the location on the new Japanese city plan map of the location determined by Hubbell on the Army map is given by using the full first-order transformation developed in this work. Hubbell himself made similar observations about the nature of the linear processes involved in his alignment of the maps in his 1986 report, noting that simply transforming the location of the hypocenter is mathematically equivalent to transforming all of the locations from which it was originally triangulated and then re-doing the triangulation on the new map. However, there are *bona fide* questions based on plane geometry that can be asked about the implications of the idea that Hubbell and others were using a U.S. Army map whose features on average were a bit too close together in the east-west direction.

One such question related to the scale of the U.S. Army map in the east-west direction has to do with triangulations based on fixed angles from particular sites. The azimuthal angle from each location to the hypocenter, based on shadows, is determined in the field and then a line is drawn on the map at that location. As it turns out, plane geometry and simple considerations based on similar triangles suggest that a misspecification of the distance in the X direction would result in an error in the Y direction when triangulations were done. The situation is illustrated in Figure 13. Although a careful reevaluation of the original work in Hubbell et al. (1969) might reveal that the Y coordinate of the hypocenter should be changed by a small amount, the question of a displacement in the north-south direction has little to do with the adjustments being suggested here, which are predominantly in an east-west direction.

Another important question, not related to the scale overall, is that any serious misplacement of particular features on the U.S. Army map where shadows were measured may have resulted in errors in placing the hypocenter in the context of that map. Without a detailed reevaluation of the original work by Hubbell and others, it is not possible to know the extent to which contributing estimates of the hypocenter location by various investigators would be in error because they relied on reference points that were particularly misplaced on the U.S. Army map. It should be noted in this regard that the errors in the placement of the Aioi Bridge, Motoyasu Bridge, A-bomb Dome, and Shima Hospital, which are discussed above, do not greatly affect the location of the hypocenter on the Army map. Most of these particular landmarks near the hypocenter were not included in the locations where shadows were measured; rather, the hypocenter location was determined by triangulation from many other points on the Army map, the relative positions of which are preserved by the transformation of the georeferencing.

These problems should not affect the decision whether or not to use the stretched image of the U.S. Army map to determine the location of the hypocenter on the new Japanese city plan maps for this work. Any change in the hypocenter location on the U.S. Army map due to inaccuracies in its aspect ratio (the horizontal stretching) or its placement of certain features could only be determined by a very detailed reassessment of the original work by Hubbell et al. (1969). There is presently no indication that removing the stretching would improve the accuracy of the hypocenter transfer to the newer maps, and the effect of the stretching is small in any case.

Additional reassurance is provided by the location determined by Kimura and Tajima, as reported by Hubbell et al. (1969), which was felt to be one of the best estimates. Kimura and Tajima reported their location as being about 25 m southeast of the main entrance to Shima Hospital. The location suggested here is about 22 m south-southeast of the main entrance to Shima Hospital as estimated from a georeferenced aerial photograph of the area near the hypocenter. This provides a datum independent of U.S. Army map coordinates that supports the location suggested here.

In summary, the crux of the matter is that the location of the hypocenter on the U.S. Army map has not changed, but a small improvement in the alignment of the U.S. Army map and the newer Japanese city plan map produces a change of about 11 to 15 m in the corresponding hypocenter position on the newer map, from the estimates that were made circa 1985 to 1986.

Nagasaki

The results for Nagasaki suggest a location for the hypocenter of (34.245, -25.394) in the coordinates of the new city plan maps, as corresponding to the U.S. Army map coordinates of (1293.624, 1065.936) given by Kerr and Solomon (1976). This location is about 3 m west of the location determined previously by staff members at RERF and discussed in an internal RERF report by T. Watanabe dated December 10, 1986. A map overlay of the area near the hypocenter is shown in Figure 14. The hypocenter is located in an irregular shaped lot to the north and east at the stadium.

Uncertainty

In studies of the dose to A-bomb survivors, as well as in the interpretation of sample measurements such as those considered elsewhere in this work, the primary map-related quantity of interest is almost always the distance from the hypocenter or epicenter of the bomb to the location of the survivor or measured sample. The uncertainty in such distances is never negligibly small, even for points whose exact location in some frame of reference (now, typically, the plane rectangular coordinates of the newer Japanese city plan maps) can be very accurately determined. The lower bound of the uncertainty in such distances is set by the substantial and irreducible uncertainty in the locations of the hypocenters and epicenters, with the latter including uncertainty in the height of burst (HOB). This section addresses only the uncertainty in ground

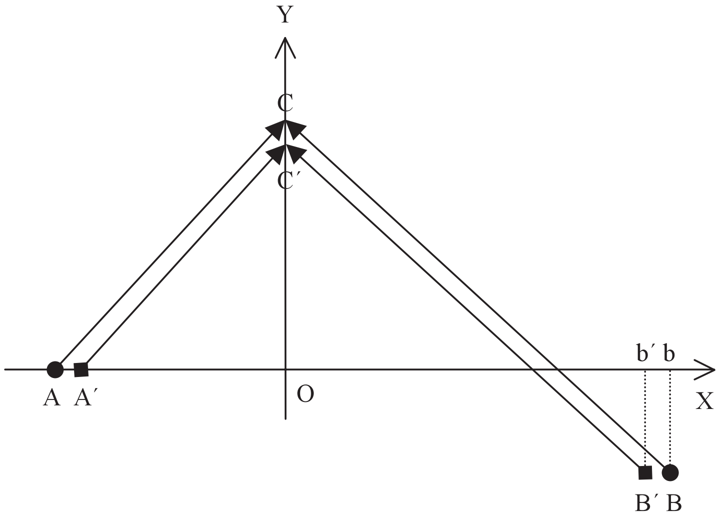

Figure 13. *An incorrect specification of the X distance between two points, with a triangulation based on fixed angles in the Cartesian grid (AC is parallel to A´C´, BC is parallel to B´C´) results in a Y displacement of the triangulated point. Note that C´ has the same placement on the X axis in relation to A´ and B´ as has C in relation to A and B; viz., in terms of segment lengths, A´O/O b´ = AO/Ob.*

Figure 14. *The area near the hypocenter in Nagasaki. The new Japanese city plan map is overlaid in green on top of the U.S. Army map. The red mark is the new suggested location for the hypocenter on the new Japanese city plan map and the green mark is the previous estimate.*

distance, i.e., distance from the hypocenter. When slant range is used, there is clearly some additional uncertainty due to the uncertainty in the height of burst.

Assuming that the uncertainty of sample and survivor distances is estimated, the treatment of this uncertainty in analyses related to the dosimetry is difficult. In analyses of measured and calculated sample values, distance is the independent variable, and error in the independent variable of statistical procedures such as regression is a very difficult statistical problem. This problem is avoided in this work largely because, although measured and calculated values of samples are often plotted vs. distance in other parts of this work, statistical procedures such as regression have typically not been used. In analyses of survivors' health outcomes, the independent variable is usually dose, which is considered a function of distance, rather than distance itself. Given these considerations, there are two major applications of interest:

1) In relation to measured samples, it is desirable to have uncertainty estimates, consistent with the idea of an error standard deviation,* that can simply be cited in publications to give the reader an idea of the size of the distance uncertainty.
2) In relation to survivors, it is desirable to have an uncertainty estimate that can be used to evaluate the contribution of distance uncertainty to the uncertainty in individual survivor doses.

The goal of this section is to provide simple formulae and estimates of related quantities that are appropriate for these purposes, using suitable approximations.

*The phrase "error standard deviation" is intended to denote the standard deviation σ of the statistical distribution of errors in the estimation of a quantity under discussion, which is assumed in this section to have an approximately normal distribution with a mean of zero, except where otherwise specified. The term "standard error," which is sometimes used in a similar way but is often not clearly defined, is avoided here in the interest of clarity, except where cited in references.

The context chosen for this work, which assumes that the newer Japanese city plan maps are highly accurate and uses them and their plane rectangular coordinate system as a fixed template, allows a systematic and comprehensive analysis of the uncertainty that affects the map-related quantities of interest. The analysis given above in previous sections supports this assumption with quantitative evaluations. It describes several types of errors that affect the *accuracy of coordinates of depicted locations on the georeferenced newer city plan maps,* as measured using the GIS:

1) error due to the approximation of the linear affine transformation vs. the full map projection equations, maximum error ~0.1 m (section on "Outline of the GIS-based Method Used for this Work"),
2) error in the relative placement of crosshair marks at intersections of the plane rectangular coordinate system (at the corners of the map tiles), used to georeference the maps, maximum error ~1 m (section on "Outline of the GIS-based Method Used for this Work," section on Georeferencing of the Newer Japanese City Plan Maps), and
3) error in the placement of features on the maps, maximum error ~1 to 2 m (section on "Outline of the GIS-based Method Used for this Work").

Regarding the first type of error listed, for this work, because the maps in the GIS are displayed in the degree space of geographical coordinates, the longitude and latitude of a depicted point on the map are read to the nearest 10^{-6} degree using a pointer provided by the GIS. Then the plane rectangular coordinates of that location are back-calculated with a solver algorithm using the full map projection equations. The first of these steps is affected by the very small error in the linear affine transformation, as that defines the longitude and latitude that the system assigns to a given pixel in the map image raster.

In addition, calculated distances between any two depicted locations on the newer city plan maps are affected by the intrinsic error due to map projection itself, which is of the order of about 2-m maximum error for the pairs of locations relevant to this work (section on "Map Projections and the Inherent Errors Associated with the Use of Plane Rectangular Coordinates").

All of these errors are small in contrast to the other errors involved in determining the locations of hypocenters, samples, and survivors, which are discussed in the following sections. They are therefore not included in the quantitative treatment set forth below. Hence, *in the following treatment, all errors are stated with respect to the plane rectangular coordinate system of the newer city plan maps, in which the features depicted on those maps are assumed for these purposes to be located with negligible error.*

Note: for ease of reference, all of the quantities representing non-negligible components of uncertainty, as used in the following discussion, are summarized in Table 13. All of the formulae derived in the following discussion are summarized in Table 14.

Uncertainty of Locations on the U.S. Army Maps

Although the present work suggests that locations and distances should be estimated directly on the newer Japanese city plan maps whenever possible, some locations can only be defined on the U.S. Army maps. Whenever a location is defined on either map by reference to features depicted on that map, the location may be a point or it may enclose some area of finite size. In the following discussion, we will refer to the former as a "well defined" location. In cases of locations that are not well defined, it is suggested that a convention be followed for purposes of consistency: a 95% confidence interval should be established for the location with respect to both coordinates, x and y, and the quoted uncertainty values in the x and y coordinates should correspond to one quarter of the width of the confidence interval, on the premise that ±2 σ is approximately the width of a 95% confidence interval for the normal distribution. This establishes a confidence ellipse for the location on the U.S. Army map, centered on the point estimate of the coordinates, as shown in Figure 15. Furthermore, it is suggested that these σ values, which we denote here as σ_{area-A_X} and σ_{area-A_Y}, be specified in meters, for ease of reference, even though the original scale of the U.S. Army map grid system is based on units of yards.

For a well defined part of a feature shown on the U.S. Army map, this uncertainty and its associated confidence ellipse may be trivially small and can be ignored. In other cases, such as samples that are known to have come from a particular graveyard or a particular building, but their location within the graveyard or building is unknown, the confidence ellipse would enclose the outline of the graveyard or building as it existed in 1945. Sometimes both the point estimate and the confidence ellipse can be better determined on a georeferenced aerial photograph, which may reduce the uncertainty.

Throughout the following treatment we will assume that errors in the X and Y coordinates are uncorrelated, which is consistent with the depiction of the ellipses in Figure 15 as having their semi-major and semi-minor axes parallel to the coordinate axes. If one thought that the area considered to contain the location of interest with 95% confidence were substantially elongated in an orientation not parallel to one of the coordinate axes, a covariance could be introduced and carried through the following steps of analysis, but that case is not discussed here.

Table 13. Quantities used in uncertainty estimates

Quantity	Units	Definition
Locations based on the U.S. Army maps		
σ_{area-A_X} , σ_{area-A_Y}	m	1/4 of the width and length of a 95% confidence ellipse bounding an area relative to features depicted on the U.S. Army map that is assumed to contain a location
σ_{A_X} , σ_{A_Y}	m	error standard deviation of the estimated coordinates of a location on the U.S. Army maps
σ_{A-0_X} , σ_{A-0_Y}	m	error standard deviation of the estimated coordinates of the hypocenters on the U.S. Army maps
$\sigma_{A-new-0_X}$, $\sigma_{A-new-0_Y}$	m	error standard deviation of the estimated coordinates of the hypocenters on the newer city plan maps as transferred from the U.S. Army maps
σ_{A-new_X} , σ_{A-new_Y}	m	error standard deviation of the estimated coordinates of a location on the newer city plan maps transferred from the U.S. Army maps
d_{01_X} , d_{01_Y}	m	X or Y component of estimated distance from the hypocenter to a location on the newer city plan map
$\sigma(d_{01_X}), \sigma(d_{01_Y})$	m	error standard deviation of the X or Y component of the distance from the hypocenter to another location
$\sigma(d_{01})$	m	error standard deviation of the distance from the hypocenter to another location
Locations based on the newer city plan maps		
$\sigma_{area-new_X}$, $\sigma_{area-new_Y}$	m	1/4 of the width and length of a 95% confidence ellipse bounding an area relative to features depicted on the newer city plan map that is assumed to contain a location
σ_{new_X} , σ_{new_Y}	m	error standard deviation of the estimated coordinates of a location on the newer city plan maps
$\sigma_{A-new-0_X}$, $\sigma_{A-new-0_Y}$	m	error standard deviation of the estimated coordinates of the hypocenters on the newer city plan maps as transferred from the U.S. Army maps
d_{01_X} , d_{01_Y}	m	X or Y component of estimated distance from the hypocenter to a location on the newer city plan map
$\sigma(d_{X_{01}}), \sigma(d_{Y_{01}})$	m	error standard deviation of the X or Y component of the distance from the hypocenter to another location
$\sigma(d_{01})$	m	error standard deviation of the distance from the hypocenter to another location