In addition to the type of uncertainty just discussed, there is a matter of uncertainty in the placement of features on a map. As discussed in the sections under “Uncertainty” above, it is assumed here that this uncertainty is negligible for the newer city plan maps, based on modern

Table 14. Recommended values and formulae for uncertainty estimates

Quantity	Formula or value to use
Locations based on the U.S. Army maps	
σ_{area-A_X} , σ_{area-A_Y}	Estimate from the U.S. Army map based on the dimensions of an area in which the point of interest is thought to be located.
σ_{A_X} , σ_{A_Y}	$\sqrt{\sigma^2_{area-A_X}+8^2}$, $\sqrt{\sigma^2_{area-A_Y}+7^2}$ (Hiroshima) $\sqrt{\sigma^2_{area-A_X}+4^2}$, $\sqrt{\sigma^2_{area-A_Y}+9^2}$ (Nagasaki)
σ_{A-0_X} , σ_{A-0_Y}	$\sigma_{A-0_X}=\sigma_{A-0_Y}=6$ m (Hiroshima) $\sigma_{A-0X}=7$ m (Nagasaki) $\sigma_{A-0_Y}=8$ m (Nagasaki)
$\sigma_{A-new-0_X}$, $\sigma_{A-new-0_Y}$	$\sigma_{A-0_X}=\sigma_{A-0_Y}=6$ m (Hiroshima) $\sigma_{A-0X}=7$ m (Nagasaki) $\sigma_{A-0_Y}=8$ m (Nagasaki)
σ_{A-new_X} , σ_{A-new_Y}	$\sqrt{\sigma^2_{area-A_X}+9^2}$, $\sqrt{\sigma^2_{area-A_Y}+8^2}$ (Hiroshima) $\sqrt{\sigma^2_{area-A_X}+5^2}$, $\sqrt{\sigma^2_{area-A_Y}+10^2}$ (Nagasaki)
d_{01_X} , d_{01_Y}	Estimate from the transformed locations on the newer city plan map.
$\sigma(d_{01_X}), \sigma(d_{01_Y})$	$\sqrt{\hat{\sigma}^2_{A-new-0_X}+\hat{\sigma}^2_{A-new_X}}$ $=\sqrt{6^2+\hat{\sigma}^2_{area-A_X}+9^2}$ (Hiroshima), $=\sqrt{7^2+\hat{\sigma}^2_{area-A_X}+5^2}$ (Nagasaki) $\sqrt{\hat{\sigma}^2_{A-new-0_Y}+\hat{\sigma}^2_{A-new_Y}}$ $=\sqrt{6^2+\hat{\sigma}^2_{area-A_X}+8^2}$ (Hiroshima), $=\sqrt{8^2+\hat{\sigma}^2_{area-A_Y}+10^2}$ (Nagasaki)
$\sigma(d_{01})$	$\sqrt{\dfrac{d^2_{0-1_X}\hat{\sigma}^2(d_{0-1_X})+d^2_{0-1_Y}\hat{\sigma}^2(d_{0-1_Y})}{d^2_{0-1_X}+d^2_{0-1_Y}}}$

Table 14. Continued

Quantity	Formula or value to use
Locations based on the newer city plan maps	
$\sigma_{area-new_X}, \sigma_{area-new_Y}$	Estimate from the newer city plan map based on the dimensions of an area in which the point of interest is thought to be located.
$\sigma_{new_X}, \sigma_{new_Y}$	$\sigma_{area-new_X}, \sigma_{area-new_Y}$
$\sigma_{A-new-0_X}, \sigma_{A-new-0_Y}$	$\sigma_{A-0_X} = \sigma_{A-0_Y} = 6$ m (Hiroshima) $\sigma_{A-0X} = 7$ m (Nagasaki) $\sigma_{A-0_Y} = 8$ m (Nagasaki)
d_{0-1_X}, d_{0-1_Y}	Estimate from the newer city plan map using hypocenter location transformed from U.S. Army map.
$\sigma(d_{0-1_X}), \sigma(d_{0-1_Y})$	$\sqrt{\hat{\sigma}^2_{A-new-0_X} + \hat{\sigma}^2_{new-1_X}} = \sqrt{6^2 + \hat{\sigma}^2_{area-new-1_X}}$ (Hiroshima), $\sqrt{7^2 + \hat{\sigma}^2_{area-new-1_X}}$ (Nagasaki) $\sqrt{\hat{\sigma}^2_{A-new-0_X} + \hat{\sigma}^2_{new-1_Y}} = \sqrt{6^2 + \hat{\sigma}^2_{area-new-1_Y}}$ (Hiroshima), $\sqrt{8^2 + \hat{\sigma}^2_{area-new-1_Y}}$ (Nagasaki)
$\sigma(d_{01})$	$\sqrt{\frac{d^2_{0-1_X}\hat{\sigma}^2(d_{0-1_X}) + d^2_{0-1_Y}\hat{\sigma}^2(d_{0-1_Y})}{d^2_{0-1_X} + d^2_{0-1_Y}}}$

map accuracy standards. For the U.S. Army maps, however, this is not the case. Modern standards for accuracy that are given in the section on "Outline of the GIS-Based Method Used for This Work" above, given the scale of the U.S. Army map, require that not more than 10% of the points tested should have an error exceeding about 10.5 m equivalent distance on the earth's surface. In absolute terms of meters on the earth's surface, this is five times larger than the criterion for the newer city plan maps because of the smaller scale of the U.S. Army maps.

These are limits, and not estimates of the actual errors in placement of features on the maps. The best available estimate of the average error in placement of features on the U.S. Army maps is the quantity $\mathbf{s}^2$ defined in the next section, "Map Transformations from the U.S. Army Maps to the Newer Japanese City Plan Maps," for the 23 locations that were able to be evaluated directly vs. the newer city plan maps. Based on the analysis documented in that section, it is recommended that an estimated error of 8-m standard deviation be considered to apply to X coordinates and an estimated error of 7-m standard deviation to Y coordinates of all locations depicted on the U.S. Army map of Hiroshima, and that an estimated error of 4-m standard deviation be considered to apply to the X coordinates and an estimated error of 9-m standard deviation to the Y coordinates of all locations depicted on the U.S. Army map of Nagasaki. These are in addition to the uncertainty in the location relative to a depicted feature or features on the

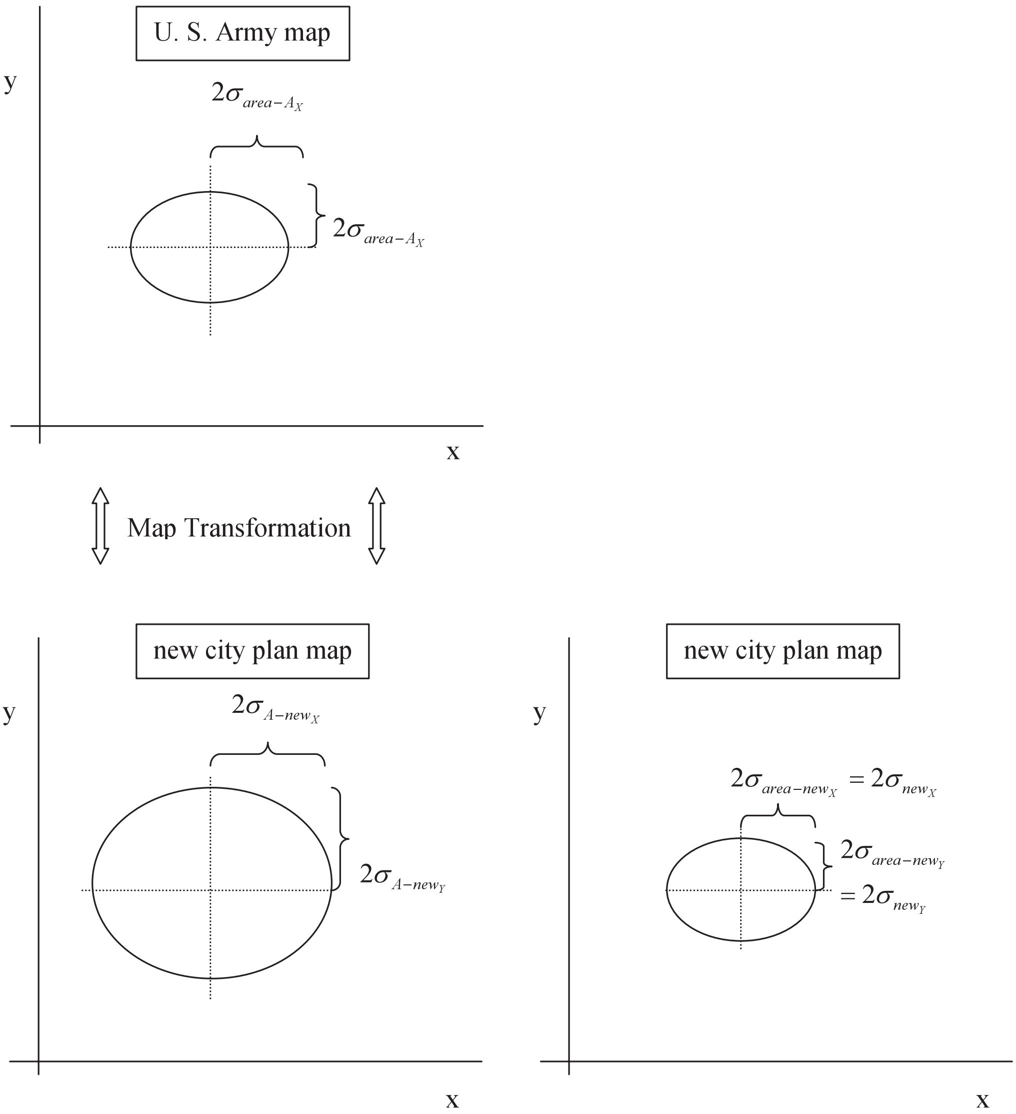

Figure 15. *Specification of a confidence ellipse enclosing "area relative to features depicted on the U.S. Army map or the newer city plan map that is assumed to contain the location of interest."*

map. Thus, the estimated size distributions of errors in the U.S. Army maps are actually consistent with modern map accuracy standards, but are not negligible for the purposes of this work, because of the relatively small scale of the U.S. Army maps.

To summarize, there are two types of errors that apply to specifying coordinates on the U.S. Army map. One has to do with the size of an area relative to features depicted on the map that is assumed to contain the sample location with 95% confidence. The other is the error in the depicted location of that area on the U.S. Army map, vs. its true location on the Army map.

Because these errors are additive and independent, the corresponding error standard deviations should be added in quadrature. For example, if the area bounding a location of interest on the U.S. Army map has an estimated σ_X of 10 m on the U.S. Army map of Hiroshima, as determined according to the concept illustrated in Figure 15, then the real σ for the X coordinate on the U.S. Army map should be estimated as $\sigma_{A_X} \cong \sqrt{\sigma^2_{area-A_X} + 8^2} = \sqrt{10^2 + 8^2} \cong 12.8$ m. The stipulation of this component of error in the context of the U.S. Army map is largely academic, as this work advocates working in the context of the newer city plan maps. It is described here primarily to make the point that it exists and to give an idea of its size.

A component of the error associated with the U.S. Army map is the error due to the finite pixel size in the scanned image raster. This is five times larger for the U.S. Army map than for the newer city plan maps due to the difference in map scale. Although very high quality scans were obtained for all of the maps, the pixel size of the U.S. Army map, as may be inferred from the scale factors B and F in Table 9, is about 7.2×10^{-6} degrees of latitude or 8.8×10^{-6} degrees of longitude per pixel, which equates to about 0.8 m/pixel in both directions when converted to meters of equivalent distance on the earth's surface. This error is small enough that it does not substantially affect the estimate of the uncertainty in U.S. Army map coordinates for the purposes of this section; however, it has a small but appreciable effect in high-precision calculations such as those discussed below in "Map Transformations from the U.S. Army Maps to the Newer Japanese City Plan Maps."

The transformation from the Army map to the newer Japanese city plan maps involves some additional uncertainty due to the transformation itself, which is addressed below in the sections on "Map Transformations from the U.S. Army Maps to the Newer Japanese City Plan Maps" and "Locations on the Newer Japanese City Plan Maps."

Locations of the Hypocenters on the U.S. Army Maps

The publications by Hubbell et al. (1969) and Kerr and Solomon (1976) give "±" values for X and Y coordinates on the U.S. Army maps. ABCC TR 3-69 (Hubbell et al. 1969) is used here as defining the correct hypocenter for Hiroshima. The table entitled "Error Estimates" on page 41 gives a value of "±16 yards or 15 m" and notes that "...it is useless to attempt to compute separate error estimates for the X and Y coordinates." Another publication, ABCC TR 23-71 (Jablon 1971) suggests that 15 m is the estimated "standard error," viz., the error standard deviation, in both the X and Y coordinates, respectively. Other investigators have understood this "±16 yards or 15 m" to represent a 95% or even a 99% confidence interval as cited in the DS86 Final Report (Kerr et al. 1987), the latter which corresponds to ±2.576 times the error standard deviation. In order to resolve this difficulty of interpretation, the data of ABCC TR 3-69, in the form of the weights assigned by the authors of that report to the various results of earlier studies that they used to obtain a combined estimate of the hypocenter location, were used to calculate the uncertainty in the X and Y coordinates of the hypocenter estimate, as described in more detail in Appendix B below. Based on this calculation, the error in the location of the Hiroshima hypocenter on the U.S. Army map is taken here as $\sigma_{A-0_X} = \sigma_{A-0_Y} = 6$ m.

Oak Ridge National Laboratory Report ORNL-TM-5139 (Kerr and Solomon 1976) is used here as defining the correct hypocenter for Nagasaki. Table 3 of ORNL-TM-5139 gives estimated σ_{A-0_X} and σ_{A-0_Y} of 7 and 8 m, respectively. Table B1 in Appendix B of ORNL-TM-5139 gives alternative values of $\sigma_{A-0_X} = 8$ m and $\sigma_{A-0_Y} = 10$ m for an alternative method of weighting the results of the individual antecedent studies on which the recommended value is based. The former values, from Table 3, 7 m and 8 m, are used here.

The estimates of hypocenter coordinates are not the same as the estimates of the true U.S. Army map coordinates of a depicted feature on the U.S. Army map. Each hypocenter estimate is a weighted average of many results of various antecedent studies based on various locations. The variability among these earlier results presumably includes the effect of the random errors in the U.S. Army map coordinates of the points used for the triangulations that produced them. *The present work therefore assumes that the estimates of* σ_X *and* σ_Y *recommended for the hypocenter coordinates by Hubbell et al. (1969) and Kerr and Solomon (1976) have already included the error arising from uncertainty in the true U.S. Army map coordinates of the individual depicted map features used in the original triangulations.* Therefore, no corresponding additional error term is added to the estimated error in the coordinates of the hypocenters on the U.S. Army maps. The values recommended by Hubbell et al. (1969) and Kerr and Solomon (1976) are summarized in Table 15.

Map Transformations from the U.S. Army Maps to the Newer Japanese City Plan Maps

The approach used here assumes that the error in a transformation from the plane rectangular coordinates of the U.S. Army maps to the plane rectangular coordinates of the newer city plan maps is dominated by the error in the transformation from pixel address in the U.S. Army map image raster to geographical coordinates of longitude and latitude. This component captures all of the error in the placement of features on the U.S. Army map relative to each other. There is a small additional error in relating the plane rectangular coordinates of the U.S. Army map to pixel address in the map image raster, to the extent that the grid lines do not form a perfectly rectilinear, Cartesian system in the pixel space of the image raster, which applies if the specification of a location on the U.S. Army map begins by measuring its plane rectangular coordinates using the grid drawn on the map, as opposed to beginning with its pixel address. This is small enough to be excluded from the following analysis, and it relates only to transformations involving the U.S. Army map grid coordinates. (Any point that can be precisely located with the GIS pointer on the image of the U.S. Army map has a pixel address that can be determined and used to calculate both longitude and latitude, and coordinates on the newer city plan maps, without using U.S. Army map grid coordinates.)

The transformation involved in georeferencing each U.S. Army map in the frame of reference

Table 15. Hypocenter locations and associated 1σ uncertainty estimates

	Hiroshima		Nagasaki	
	X	Y	X	Y
DS86 U.S. Army map[a]	744.298 kyds 6 m	1,261.707 kyds 6 m	1,293.624 kyds 7 m	1,065.936 kyds 8 m
This work: new Japanese city plan map	26.721 km 6 m	-178.395 km 6 m	34.245 km 7 m	-25.394 km 8 m

[a]The values quoted for DS86 are based on the original reports (Hubbell et al. 1969, Kerr and Solomon 1976). All of the values given here, including the σ values, are rounded to the nearest m. The Nagasaki hypocenter coordinates given by Kerr and Solomon (1976) are shown in the table; Hubbell et al.'s (1969) value was 1293.626, 1065.932.

of the corresponding georeferenced new city plan maps, using the control points chosen for features depicted on both maps, establishes a mathematical relationship between the U.S. Army map as it exists in its original image raster form and its georeferenced form. For any feature depicted on the U.S. Army map, this is the relationship between the X and Y components of the pixel address and the longitude and latitude in the primary frame of reference used in this work: the geographical coordinate system based on the Tokyo datum. The transformation is defined by the simple equations (1) and (2) in the above section entitled "Georeferencing." By definition, such a first-order affine transformation constitutes a regression model that can be used for estimating coordinate transformations and the errors associated with them. The residuals of this regression have been examined in detail to evaluate its adequacy, as documented in the section on "Results" under "Outline of the GIS-based Method Used for this Work." Thus, the uncertainty in the longitude and latitude estimated by the transformation of a pixel address in the U.S. Army map image raster is defined by the regression that estimates the fitted coefficients A through F of the transformation.

Because the GIS software fits these simple linear equations by least squares, the fitting process implicit in the GIS's georeferencing equates to a bivariate multiple linear regression using the data for the chosen control points, with the independent variable being the pixel addresses in the x and y coordinates, i.e., column and row, respectively, of the U.S. Army map image raster, and the dependent variable being the longitudes and latitudes defined by the new city plan map as the correct values for those control points. The x and y used here are the i and j in equations (1) and (2) of the above section entitled "Georeferencing." If we denote the vectors of x and y values, plus a vector of 1's for the intercept, as a 3 × n matrix **XY** and the vectors of the longitude and latitude values as a 2 × n matrix **L**, where n is the number of control points, then the estimates of the fitted coefficients A through F are given by the regression $\mathbf{L} = \mathbf{XY}\,\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, where β is the matrix of the coefficients of the first-order affine transformation and ε is the two vectors of errors in the regression:

$$\mathbf{L} = \begin{bmatrix} long_1\ lat_1 \\ long_2\ lat_2 \\ . \quad . \\ . \quad . \\ . \quad . \\ long_n\ lat_n \end{bmatrix}, \mathbf{XY} = \begin{bmatrix} 1\, x_1\ y_1 \\ 1\, x_2\ y_2 \\ 1\ .\ . \\ 1\ .\ . \\ 1\ .\ . \\ 1\, x_n\ y_n \end{bmatrix}, \mathbf{b} = \hat{\boldsymbol{\beta}} = \begin{bmatrix} A\,D \\ B\,E \\ C\,F \end{bmatrix}, and\ \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11}\ \varepsilon_{12} \\ \varepsilon_{21}\ \varepsilon_{22} \\ . \quad . \\ . \quad . \\ . \quad . \\ \varepsilon_{n1}\ \varepsilon_{n2} \end{bmatrix} \tag{4}$$

This regression was checked by the "mvreg" multivariate regression program of Stata™ 7.1 (Stata Corporation 2001) and also by simply calculating the solution $\hat{\beta} = (XY'XY)^{-1}\ XY'L$ in MatLab™6.1 (The Mathworks, Inc. 2001). An excellent reference for regression is *Applied Regression Analysis* by Draper and Smith (1981).

The input values for the regression for Hiroshima are shown in Table 16. The longitude and latitude values for the control points are easily obtained from the GIS by using a software tool that it provides. Because the GIS tool does not display the pixel address of the selected location, to obtain the most accurate values for the pixel addresses of the control points that were actually used, the values were back-calculated from the fitted coefficients A through F by solving the simple system of equations (1) and (2) for i and j, which are the individual values in the **XY** of the regression in equation (3). These were checked by visual inspection of the image raster via an image editing software package that provides a tool for directly determining the pixel address of a point indicated by the cursor on the displayed image.

The regression used by the GIS is in the wrong direction, however, for a proper statistical analysis. Conventional linear regression is based on ignoring the errors in the independent variable, which are assumed to be negligible, and associating the modeled error with the dependent variable. The non-trivial errors in this problem are associated with the pixel addresses of the control points and not with their longitude and latitude, as the latter are accurately known from the newer city plan maps. The proper statistical approach is therefore to regress pixel address on longitude and latitude and then take the inverse of the fitted parameters as the matrix defining the transformation for georeferencing. Hence, we should take

$$\mathbf{XY} = \begin{bmatrix} x_1 \; y_1 \\ x_2 \; y_2 \\ . \quad . \\ . \quad . \\ . \quad . \\ x_n \; y_n \end{bmatrix}, \mathbf{L} = \begin{bmatrix} 1 & long_1 & lat_1 \\ 1 & long_2 & lat_2 \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & long_n & lat_n \end{bmatrix}, \mathbf{b}_{inv} = \hat{\boldsymbol{\beta}}_{inv} = \begin{bmatrix} A_{inv} \; D_{inv} \\ B_{inv} \; E_{inv} \\ C_{inv} \; F_{inv} \end{bmatrix}, and\ \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} \; \varepsilon_{12} \\ \varepsilon_{21} \; \varepsilon_{22} \\ . \quad . \\ . \quad . \\ . \quad . \\ \varepsilon_{n1} \; \varepsilon_{n2} \end{bmatrix} \quad (5)$$

set up the regression as $\mathbf{XY} = \mathbf{L}\beta + \varepsilon$, and invert the fitted parameters to obtain the transformation that is appropriate for converting XY coordinates to longitude and latitude, i.e., an inverse regression.

Table 16. Input values for the multiple linear regression of longitude and latitude vs. map image raster pixel address for the 23 control points in Hiroshima

Control Point	Longitude (degrees)	Latitude (degrees)	Pixel X	Address Y
Motoyasu Bridge, mid-river	132.456179	34.390844	7220	5013
Center of dome of A-bomb Dome	132.456065	34.392230	7211	4814
Fukoku Seimei Bldg, SW corner	132.459331	34.388878	7585	5291
Teikoku Bank, NE corner (Andersen's)	132.461102	34.389914	7790	5146
Bank of Japan Hiroshima Branch, SW corner	132.459094	34.388147	7553	5398
Honkawa Bridge, mid-river	132.453063	34.390493	6860	5070
Naka Telephone Bldg, SW corner	132.461542	34.387912	7832	5424
San'in Godo Bank, NW corner	132.464073	34.390862	8122	5021
Kirin Beer Hall, NE corner	132.464343	34.389065	8155	5263
Chuden Bldg, SW corner	132.458066	34.385113	7441	5812
Fukuya Dept. Store, NW corner	132.464950	34.390619	8220	5055
Hiroshima Castle moat, SW outside corner	132.459853	34.397435	7668	4094
Kodo National School, center	132.449317	34.392126	6466	4859
Yorozuyo Bridge, mid-river	132.453079	34.384208	6872	5950
City Hall, SW corner	132.457164	34.381873	7332	6257
Hiroshima Telephone Co. West Branch SW corner	132.446319	34.394939	6121	4454
Meiji Bridge, mid-river	132.452924	34.380432	6841	6469
Yokogawa Bridge, mid-river	132.452733	34.402295	6825	3403
Kyo Bridge, mid-river	132.472453	34.390955	9094	4984
Red Cross Hospital, NW corner	132.457076	34.378231	7331	6766
University "E" Bldg, NW corner	132.460667	34.379065	7718	6650
Sakae Bridge, mid-river	132.473317	34.395115	9195	4420
Hijiyama Bridge, W end at river bank	132.469763	34.380382	8780	6457

Fortunately, both the forward and inverse regressions give almost identical results in the problem of interest here. The results of both forward and inverse regression are shown in Table 17, along with the percent difference in parameter estimates. These are identical to the values in the world file (.tfw file) created by the GIS, shown in Table 18, except for a small difference that appears to be due to round-off errors and the finite pixel size of the scanned U.S. Army map image. The scanned raster of the U.S. Army map has pixels that are about 0.8 m × 0.8 m in terms of equivalent map distance, as discussed in the section on "Uncertainty of Locations on the U.S. Army Maps" above. The small differences between the coefficient values in Tables 17 and 18 correspond to small errors with a mean of about 1.4 m among the locations of the 23 points used for map alignment.

For the regression $\mathbf{L} = \mathbf{XY}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, to obtain the estimated error in the longitude and latitude $\mathbf{L}_0$ calculated from the X and Y coordinates $\mathbf{XY}_0$ of some new point not included in the regression, the formula is

$$\mathrm{var}(\hat{\mathbf{L}}_0) = \mathbf{s}^2[1 + \mathbf{XY}_0'(\mathbf{XY}'\mathbf{XY})^{-1}\mathbf{XY}_0] \quad (6)$$

where $\mathbf{s}^2$ contains one entry for longitude and one for latitude, and is the mean square error associated with the residual sum of squares of the regression. The sum of squares of the differences between the true values of longitude and latitude for the control points in **L** and those predicted by the fitted regression coefficients for the pixel addresses of those points, divided by the number of control points minus the 3 degrees of freedom used in the regression for the

Table 17. Results of multivariate linear regression of longitude and latitude vs. map image raster pixel address for the 23 control points in Hiroshima

	Coefficient	Estimate	Std error	95% confidence interval	
Longitude	constant (A)	132.392634	2.30×10^{-4}	132.3921	132.3931
	x (B)	8.7568×10^{-6}	2.61×10^{-8}	8.70×10^{-6}	8.81×10^{-6}
	y (C)	5.39×10^{-8}	2.41×10^{-8}	3.56×10^{-9}	1.04×10^{-7}
Latitude	constant (D)	34.42713	1.53×10^{-4}	34.42681	34.42744
	x (E)	-3.5187×10^{-8}	1.73×10^{-8}	-7.15×10^{-8}	6.05×10^{-10}
	y (F)	-7.1813×10^{-6}	1.60×10^{-8}	-7.22×10^{-6}	-7.15×10^{-6}

Table 18. Fitted parameter values produced by georeferencing of U.S. Army map using 23 control points, as recorded in .tfw ("World") file for Hiroshima

	Coefficient	Estimate
Longitude	constant (A)	132.39258
	x (B)	0.00000876007
	y (C)	0.000000054954
Latitude	constant (D)	34.42714
	x (E)	-0.000000036483
	y (F)	-0.0000071835

intercept and the fitted coefficients for X and Y give:

$$\mathbf{s}^2 = \frac{\mathbf{L'L} - \mathbf{b'XY'L}}{n-3} \tag{7}$$

The first term inside the brackets in equation (6), $\mathbf{s}^2$ is the "error about the regression line," as it is typically considered in regression. The second term, $\mathbf{s}^2\mathbf{XY'_0(XY'XY)^{-1}XY_0}$, is the "error in the regression line." The estimate of the uncertainty in the difference between the fitted line and the true line at the location of interest is much smaller than the error about the regression line.

For Hiroshima, $\mathbf{s_{long}}$ is about 9.2×10^{-5} degrees and $\mathbf{s_{lat}} \cong 6.4 \times 10^{-5}$ degrees. These can be approximated in meters in the X and Y directions by ignoring the small rotation of the map coordinate X and Y vs. the X and Y of the image rasters, and using a value of 111,111 m per degree of latitude based on the simple spherical earth approximation, and 111,111[cos (lat)] m per degree of longitude. The results are about 8.4 m in the east-west direction and 7.1 m in the north-south direction.

For Nagasaki, $\mathbf{s_{long}}$ is about 4.3×10^{-5} degrees and $\mathbf{s_{lat}} \cong 8.4 \times 10^{-5}$ degrees, which approximate to 4.0 m in the east-west direction and 9.3 m in the north-south direction.

For constructing a confidence interval, these values should really be used with the points of a t distribution rather than a standard normal distribution, because the values of s have been estimated from the same data as were used to estimate the transformation coefficients **b**. However, the number of degrees of freedom involved here is large enough that the confidence intervals are not much wider. For example, a 95% confidence interval would be ±2.086s for the t distribution with n - 3 = 20 degrees of freedom, as opposed to ±1.96s for the normal distribution.

If we consider the more correct approach of taking the regression as $\mathbf{XY = L\boldsymbol{\beta} + \boldsymbol{\varepsilon}}$ and using its inverse to define **L** as a function of **XY**, the question that arises is how to calculate the uncertainty in a value $\mathbf{L_0}$ that corresponds to some new point $\mathbf{XY_0}$ in the map image raster that was not used in the regression. The confidence intervals for the inverse regression are derived in Appendix C, and are shown to differ from the confidence intervals obtained here from the regression $\mathbf{L = XY\boldsymbol{\beta} + \boldsymbol{\varepsilon}}$ by only a very small amount; therefore, the values derived here from the forward regression $\mathbf{L = XY\boldsymbol{\beta} + \boldsymbol{\varepsilon}}$ are recommended for use.

For the locations very near the hypocenter in Hiroshima, equation (5) results in an estimated error of about 9.4×10^{-5} degrees of longitude and 6.5×10^{-5} degrees of latitude, or about 8.6 m in the X direction and 7.3 m in the Y direction. This estimated error increases slightly with distance from the center of the control points. For instance, the value for longitude at the location of the Hijiyama Bridge equates to about 9.3 m rather than 8.6 m.

For Nagasaki, the scatter of the control points about the regression in the Y (north-south) direction is considerably larger than in the X direction, and the estimated error of the transformed coordinates for a point at the hypocenter location is only about 4.4 m in the X direction, but is about 9.9 m in the Y direction.

It is seen that the transformation itself adds relatively little to the uncertainty in the newer city plan map coordinates of a location depicted on the U.S. Army map: typically, most of the uncertainty is due to the errors in placement of the features on the U.S. Army map, and not due to the uncertainty in the transformation coefficients.

Based on these estimates, it is recommended that locations transformed from the U.S. Army maps to the newer city plan maps in Hiroshima be considered to have estimated errors calculated by adding 9 m in quadrature to the X component and 8 m to the Y component of the error estimated by drawing a confidence ellipse on the U.S. Army map. For example, if the area bounding a location of interest on the U.S. Army map has an estimated σ_X and σ_Y of 10 m on the U.S. Army map, as determined according to the concept illustrated in Figure 15, then the real σ for the error in the X coordinate on the newer city plan map should be estimated as $\sigma_{A-new_X} \cong \sqrt{\sigma^2_{area-A_X} + 9^2} = \sqrt{10^2 + 9^2} \cong 13.5$ m, and $\sigma_{A-new_Y} \cong \sqrt{\sigma^2_{area-A_Y} + 8^2} \cong 12.8$. These are slightly larger than σ_{A_X} and σ_{A_Y} because of the uncertainty in the transformation. For Nagasaki, it is recommended that the formulae $\sigma_{A-new_X} \cong \sqrt{\sigma^2_{area-A_X} + 5^2}$ and $\sigma_{A-new_Y} \cong \sqrt{\sigma^2_{area-A_Y} + 10^2}$ be used.

Locations on the Newer Japanese City Plan Maps

A straightforward way to express the uncertainty in locations based on features depicted on the newer city plan maps is to use the same convention established in the section above on "Uncertainty of Locations on the U.S. Army Maps." For estimating a standard deviation of the error in the X and Y coordinates, σ_X and σ_Y, respectively, one quarter of the appropriate dimension is used of an ellipse that encloses the area in which the location of interest is known to be included with 95% confidence. Using the same sort of notation, $\sigma_{new_X} = \sigma_{area-new_X}$ and $\sigma_{new_Y} = \sigma_{area-new_Y}$, where $\sigma_{area-new_X}$ and $\sigma_{area-new_Y}$ are determined directly on the newer city plan maps, and may be taken as zero for cases of well defined locations depicted on those maps. Here, there is no additional uncertainty associated with map transformation or with the error in the placement of features on the map, because of the accuracy of the newer city plan maps.

As depicted in Figure 15, if the location can only be determined on the U.S. Army map, then the error standard deviation on the newer city plan map should be estimated as

$$\hat{\sigma}_{A-new\,X} = \sqrt{\hat{\sigma}^2_{area-A_X} + 9^2} \quad \textit{and} \quad \hat{\sigma}_{A-new_X} = \sqrt{\hat{\sigma}^2_{area-A_Y} + 8^2} \quad (\textit{Hiroshima})$$
$$\hat{\sigma}_{A-new_X} = \sqrt{\hat{\sigma}^2_{area-A_X} + 5^2} \quad \textit{and} \quad \hat{\sigma}_{A-new_Y} = \sqrt{\hat{\sigma}^2_{area-A_Y} + 10^2} \quad (\textit{Nagasaki}) \qquad (8)$$

as discussed in the preceding section, wherein σ_{area-A_X} and σ_{area-A_Y} are determined on the U.S. Army maps.

In using this formulation, we ignore any covariance between the errors made in determining the location of a point on the Army map, and the error in determining the map transformation. Of course, the errors would be correlated for any locations specified on the U.S. Army map that are based on any of the 23 control points used in determining the map transformation. However, this is not an issue because those 23 points were chosen because they were also depicted on the newer city plan maps, and the latter should be used rather than the former; i.e., no map transformation is necessary for locations based on those features.

Locations of the Hypocenters on the Newer City Plan Maps

The locations of the hypocenters on the newer city plan maps clearly constitute a case in which the determination cannot be made directly on the newer city plan maps. Hence, the errors associated with the map transformation as discussed above in the sections "Map Transformations from the U.S. Army Maps to the Newer Japanese City Plan Maps" and "Locations on the Newer

Japanese City Plan Maps" apply. Based on the approach articulated above under "Locations on the Newer Japanese City Plan Maps," a reasonable way to calculate the resulting uncertainty would be to use equation (6) and replace the longitude and latitude components of $\mathbf{s}^2$ in m with the values of $\sigma^2_{A-0_X}$ and $\sigma^2_{A-0_Y}$ for the hypocenter, respectively, given in the first row of Table 15. This results in the recommended values shown in the second row of Table 15, which are identical to those in the first row when rounded to the nearest meter, because the square root of the expression in brackets in (6) evaluates to about 1.03 at the hypocenter in Hiroshima and 1.06 at the hypocenter in Nagasaki. That is, for the purposes of this section, we round to the nearest m and simply use $\sigma_{A-new-0_X} \cong \sigma_{A-0_X}$ and $\sigma_{A-new-0_Y} \cong \sigma_{A-0_Y}$.

This method assumes that there is no correlation between the error in specifying the hypocenter location on the U.S. Army map and the errors in the coefficients for the map transformation. Although it is possible that errors in the depicted locations of some key landmarks on the U.S. Army map that were involved in both processes might have caused somewhat correlated errors in both the hypocenter location and the fitted map transformation, this is expected to be a small contribution and is not practical to estimate in this work. In Hiroshima, for instance, the vast majority of the influential measurements for the hypocenter triangulations in the individual studies used in ABCC TR 3-69 (Hubbell et al. 1969) were made at cemeteries (Arakawa and Nagaoka study, Woodbury and Mizuki study) or other locations such as the Gokoku Shrine, Sino-Japanese War Monument, Chamber of Commerce (Kimura and Tajima study) that are not among the landmarks used for georeferencing the U.S. Army map in this work.

Distances from the Hypocenters to Other Locations

This problem has been considered before, but using a different approach, and in a situation based entirely in the context of U.S. Army map coordinates: Jablon (1971) gave a derivation based on trigonometry. In addition to having no need to consider a transformation of coordinates to another map, Jablon made two other assumptions:

1) His work was oriented strictly toward survivor distance, and he treated the issue of uncertainty in the survivor's location separately; hence the portion of his work that is considered here addresses only the effect of the uncertainty in the location of the hypocenter on the uncertainty in the distance to some fixed survivor location. He called this distance "d."
2) Consistent with the position of ABCC TR 3-69 (Hubbell et al. 1969) that separate uncertainty estimates for the X and Y coordinates of the hypocenter were not feasible, he based his derivation on the distance from the hypocenter's true location to its estimated location, which he denoted "r" not on separate errors in the X and Y coordinates, and made assumptions about the geometry and statistical distribution of the vector associated with that distance that are consistent with equal and independent error distributions in the X and Y coordinates of the hypocenter (i.e., that the error distribution has circular symmetry about the hypocenter).

He then calculated the ratio of the error standard deviation of d, σ_d, to the error standard deviation of r, σ_r, for points far enough away that d is much larger than σ_r. In Appendix C, the relationship between error specified in polar coordinates and error specified in Cartesian (X-Y) coordinates is shown, and Jablon's result is compared to the result of this work. The ratio suggested by this work is $\frac{\sigma_d}{\sigma_r} = \sqrt{\frac{2}{4-\pi}} \cong 1.53$ for the case in which the error has circular symmetry (i.e., uncorrelated errors in the X and Y coordinates meet this criterion of circular symmetry if they have identical distributions).

In this work, we want to consider both the uncertainty in the coordinates of the hypocenter and the uncertainty in the coordinates of the other location, whether it applies to a sample or a survivor. We also want to allow for the error in the X coordinate to have a different mean than the error in the Y coordinates, and we must consider the issue of map transformation. Therefore, we use a component-wise approach that is consistent with the previous sections, explicitly considering the errors in X and Y coordinates of both the hypocenter and the other location to which we wish to estimate the distance.

The relationship is simple, but is fraught with possibilities for correlated errors in several respects, which must be carefully considered. If we denote the difference between the X coordinate X_0 of the hypocenter and the X coordinate X_1 of some other point as d_{01_X} , viz., $d_{01_X} = X_1 - X_0$, then by the definitions of variance and covariance, we can derive the simple formulation

$$\hat{\sigma}(d_{0-1_X}) = \sqrt{\hat{\sigma}_{0_X}^2 + \hat{\sigma}_{1_X}^2 - 2\hat{\sigma}_{0-1_X}} \tag{9}$$

where $\hat{\sigma}_{0-1_X}$ is the covariance of X_0 and X_1, and it is important to note the minus sign associated with the covariance term. There are two reasons why the covariance term might be nonzero:

1) For locations that can only be determined on the U.S. Army map, and that are based on the locations that were used originally to determine the hypocenter (Hubbell et al. 1969), there might be a correlation between the error in the depiction of the related feature on the U.S. Army map and the error in the location of the hypocenter.
2) For any location that can only be determined on the U.S. Army map, the same transformation is used to place both that location and the hypocenter on the new city map, hence the errors due to the transformation for the hypocenter and the other point are systematically related.

The covariance due to 1) appears to be impractical to estimate and not of much general concern, because the hypocenter location is based on triangulations from many points. Thus, the correlation of the hypocenter error with the error in any individual feature depicted on the U.S. Army map is likely to be very small.

In considering the covariance due to 2), we must go back to the level of the regression relating U.S. Army map pixel address and geographical coordinates of longitude and latitude, as we did in deriving the uncertainty estimates in previous sections. For each of longitude and latitude, there are then three parameter estimates for the linear transformation: either A, B, and C; or D, E, and F. The errors in these parameters must be considered in addition to the errors in the specification of the pixel address (or, equivalently for our purposes, the U.S. Army map grid coordinates) of the hypocenter and the other feature to which we are estimating the distance. Fortunately, reasonable assumptions allow us to ignore the covariance among these quantities, except for the covariance among the parameter estimates, as follows:

1) We assume that the errors in the location on the U.S. Army map of the hypocenter and the location of any other feature depicted on the U.S. Army map are stochastically independent, for the reasons just stated in dismissing 1) above.
2) We also assume that the errors in the transformation coefficients are stochastically independent of the errors in the pixel address of the hypocenter, although there might be some very small covariance to the extent that the hypocenter coordinates were determined using locations based on a few of the same 23 features on the U.S. Army map that were used to

estimate the transformation coefficients, as discussed in detail at the end of the preceding section on "Locations of the Hypocenters on the Newer City Plan Maps."

3) Finally, we assume that the errors in the transformation coefficients are stochastically independent of the errors in the location of any other feature depicted on the U.S. Army map, to which we want to measure the distance from the hypocenter. This is not strictly true for the 23 features on the U.S. Army map that were used to estimate the transformation coefficients, of course. However, as pointed out in the section "Locations on the Newer Japanese City Plan Maps" above, this concern does not arise, because these 23 locations are depicted on the newer city plan maps; hence they do not need to be transformed from the U.S. Army map.

Thus, if the transformation from pixel address is, e.g., $longitude = A + Bi + Cj$, wherein i and j are the row and column of the pixel address of the feature of interest in the image raster of the U.S. Army map, and we consider the joint probability density function of the transformation coefficients A, B, and C and the pixel address components i and j, our assumptions about stochastic independence give

$$f(A,B,C,i,j;\theta) = f_{ABC}(A,B,C;\theta_{ABC}) f_i(i;\theta_i) f_j(j;\theta_j) \tag{10}$$

wherein θ is a vector of parameters (i.e., the means and standard deviations of a multivariate normal distribution). With these assumptions we can show that the covariance terms affecting the error in longitude and latitude, and consequently the covariance term in equation (9), is negligibly small in contrast to the other terms in (9). A derivation and example calculation are given in Appendix D.

For distances from the hypocenter *to locations that can only be determined on the U.S. Army map*, we could rewrite equation (9) to a good approximation as

$$\hat{\sigma}(d_{0-1_X}) \cong \sqrt{\hat{\sigma}^2_{A-new-0_X} + \hat{\sigma}^2_{A-new-1_X}} \quad \hat{\sigma}(d_{0-1_Y}) \cong \sqrt{\hat{\sigma}^2_{A-new-0_Y} + \hat{\sigma}^2_{A-new-1_Y}} \tag{11}$$

$$\hat{\sigma}(d_{0-1_X}) \cong \sqrt{\hat{\sigma}^2_{A-new-0_X} + \hat{\sigma}^2_{new-1_X}} \quad and \quad \hat{\sigma}(d_{0-1_Y}) \cong \sqrt{\hat{\sigma}^2_{A-new-0_Y} + \hat{\sigma}^2_{new-1_Y}} \tag{12}$$

with $\hat{\sigma}_{A-new-0_X}$ and $\hat{\sigma}_{A-new-0_Y}$ being estimated for the hypocenter as in "Locations of the Hypocenters on the Newer City Plan Maps" above and Table 15, and $\hat{\sigma}_{new-1_X}$ and $\hat{\sigma}_{new-1_Y}$ being estimated for the point of interest directly on the newer city plan map, in m, as in the first part of "Locations on the Newer Japanese City Plan Maps" above.

Since the distance $d_{0\text{-}1_X}$ is defined by the simple relationship $d_{0-1} = \sqrt{d^2_{0-1_X} + d^2_{0-1_Y}}$, the formulae for estimating propagation of error based on first-order Taylor series approximations (Bevington and Robinson 1992) can be used by successively applying the power formula $v = u^a \Rightarrow \frac{\sigma_v}{v} \cong a\frac{\sigma_u}{u}$, the sum formula for independent quantities $w = u + v \Rightarrow \sigma_w \cong \sqrt{\sigma_u^2 + \sigma_v^2}$, and the power formula again, as

$$\frac{\sigma(d^2_{0-1_X})}{d^2_{0-1_X}} \cong 2\frac{\sigma(d_{0-1_X})}{d_{0-1_X}} \quad \Rightarrow \quad \sigma(d^2_{0-1_X}) \cong 2d_{0-1_X}\sigma(d_{0-1_X}) \tag{13}$$

Next Page