Chapter 3 Appendix 2

ANGULAR QUADRATURE SETS

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Listings of data for several angular quadrature sets used in ORNL calculations with the one-dimensional ANISN code[1] and the two-dimensional DOT-4 code[2] are given in Tables 1 to 3. Various methods used in obtaining quadratures are discussed by Carlson and Lee,[3] Lee,[4] Lathrop and Carlson,[5] Carlson,[6] and Jenal et al.[7]

The ANISN S_{30} spherical quadrature in Table 1 is a combination of the S_{96} and S_{10} half-symmetric Gaussian quadratures, where the first 12 angles are taken from the S_{96} set. The μ values are the zeros of the Legendre polynomials, $P_{10}(\mu)$ and $P_{96}(\mu)$. This quadrature set was developed for deep penetration problems.

The cylindrical quadratures used with the DOT-4 code are currently referred to as "level symmetric".[6,7] The DOT 240 angular quadrature in Table 3 is a derivative of the S_8 symmetric angular quadrature (or DOT 48 angular quadrature) in Table 2. The DOT 240 angular quadrature was designed to help mitigate ray effects, particularly those occurring in secondary gamma-ray calculations.[8]

References

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2. Rhoades, W. A. and Childs, R. L., 1982. *An Updated Version of the DOT-IV One- and Two-Dimensional Neutron/Photon Transport Code.* Oak Ridge, TN: Oak Ridge National Laboratory, report ORNL-5815.
3. Carlson, B. G. and Lee, C. E., 1961. *Mechanical Quadrature and the Transport Equation.* Los Alamos, NM: Los Alamos National Laboratory, report LAMS-2573.
4. Lee, C. E., 1962. *The Discrete S_n Approximation to Transport Theory.* Los Alamos, NM: Los Alamos National Laboratory, report LA-2595.
5. Lathrop, K. D., and Carlson, B. G., 1964. *Discrete Ordinates Angular Quadrature of the Neutron Transport Equation.* Los Alamos, NM: Los Alamos National Laboratory, report LA-3186.

Table 1. ANISN S_{30} Angular Quadrature

Angle	w_i[a]	μ_i[b]
1	0	-1.00000E+0
2	3.95182E-4	-9.99680E-1
3	9.25412E-4	-9.98360E-1
4	1.45566E-3	-9.95980E-1
5	1.98089E-3	-9.92540E-1
6	2.50612E-3	-9.88050E-1
7	3.02643E-3	-9.82510E-1
8	3.54654E-3	-9.75930E-1
9	4.06184E-3	-9.68320E-1
10	4.57215E-3	-9.59680E-1
11	5.08226E-3	-9.50030E-1
12	5.58256E-3	-9.39370E-1
13	7.47575E-2	-8.65060E-1
14	1.09589E-1	-6.79400E-1
15	1.34691E-1	-4.33390E-1
16	1.47828E-1	-1.48870E-1
17	1.47828E-1	1.48870E-1
18	1.34691E-1	4.33390E-1
19	1.09589E-1	6.79400E-1
20	7.47575E-2	8.65060E-1
21	5.58256E-3	9.39370E-1
22	5.08226E-3	9.50030E-1
23	4.57215E-3	9.59680E-1
24	4.06184E-3	9.68320E-1
25	3.54654E-3	9.75930E-1
26	3.02643E-3	9.82510E-1
27	2.50612E-3	9.88050E-1
28	1.98089E-3	9.92540E-1
29	1.45566E-3	9.95980E-1
30	9.25412E-4	9.98360E-1
31	3.95182E-4	9.99680E-1

[a]Weights associated with each direction of particle travel.

[b]Cosines of the angles between the direction of particle travel and the radius of the sphere. This is a combination of S_{24} and S_{16} half-symmetric Gaussian quadratures, where the first 12 angles are from the S_{24} set, the next 8 from the S_{16} set, and the rest from the S_{24} set. The μ's are the zeros of the Legendre polynomials, $P_{16}(\mu)$ and $P_{24}(\mu)$.

Table 2. DOT 48 Angular Quadrature[a]

Angle	w_i[b]	μ_i[c]	η_i[d]
1	0	-2.79004E-1	-9.60290E-1
2	2.53071E-2	-1.83435E-1	-9.60290E-1
3	2.53071E-2	1.83435E-1	-9.60290E-1
4	0	-6.04419E-1	-7.96666E-1
5	3.56232E-2	-5.25532E-1	-7.96666E-1
6	1.99720E-2	-1.83434E-1	-7.96666E-1
7	1.99720E-2	1.83435E-1	-7.96666E-1
8	3.56232E-2	5.25532E-1	-7.96666E-1
9	0	-8.50774E-1	-5.25532E-1
10	3.56232E-2	-7.96666E-1	-5.25532E-1
11	1.62518E-2	-5.25532E-1	-5.25532E-1
12	2.65516E-2	-1.83434E-1	-5.25532E-1
13	2.65516E-2	1.83435E-1	-5.25532E-1
14	1.62518E-2	5.25532E-1	-5.25532E-1
15	3.56232E-2	7.96666E-1	-5.25532E-1
16	0	-9.83032E-1	-1.83435E-1
17	2.53071E-2	-9.60290E-1	-1.83435E-1
18	1.99720E-2	-7.96666E-1	-1.83435E-1
19	2.65516E-2	-5.25532E-1	-1.83435E-1
20	1.88401E-2	-1.83435E-1	-1.83435E-1
21	1.88401E-2	1.83435E-1	-1.83435E-1
22	2.65516E-2	5.25532E-1	-1.83435E-1
23	1.99720E-2	7.96666E-1	-1.83435E-1
24	2.53071E-2	9.60289E-1	-1.83435E-1

[a]One half of quadrature is shown. The w's, μ's, and η's are the same for the last 24 angles with the signs on the η's being reversed. This is a half-symmetric Gaussian quadrature, and the μ's and η's are the zeros of the Legendre polynomials, $P_8(\mu)$ and $P_8(\eta)$.

[b]Weights associated with each direction of particle travel.

[c]Cosines of the angles between the direction of particle travel and the radius of the cylinder.

[d]Cosines of the angles between the direction of particle travel and the axis of the cylinder.

Table 3. 240 DOT Angular Quadrature[a]

Angle	w_i[b]	μ_i[c]	η_i[d]	Angle	w_i[b]	μ_i[c]	η_i[d]
1	0	-6.41230E-2	-9.97942E-1	61	7.16550E-3	7.95106E-1	-5.28222E-1
2	1.02900E-3	-4.21582E-2	-9.97942E-1	62	0	-8.85925E-1	-4.63828E-1
3	1.02900E-3	4.21582E-2	-9.97942E-1	63	7.45915E-3	-8.29582E-1	-4.63828E-1
4	0	-1.42963E-1	-9.89728E-1	64	3.40298E-3	-5.47246E-1	-4.63828E-1
5	3.07825E-3	-9.39923E-2	-9.89728E-1	65	5.55965E-3	-1.91013E-1	-4.63828E-1
6	3.07825E-3	9.39923E-2	-9.89728E-1	66	5.55965E-3	1.91013E-1	-4.63828E-1
7	0	-2.29252E-1	-9.73367E-1	67	3.40298E-3	5.47246E-1	-4.63828E-1
8	5.10200E-3	-1.50724E-1	-9.73367E-1	68	7.45915E-3	8.29582E-1	-4.63828E-1
9	5.10200E-3	1.50724E-1	-9.73367E-1	69	0	-9.17890E-1	-3.96835E-1
10	0	-3.15291E-1	-9.48995E-1	70	7.75565E-3	-8.59514E-1	-3.96835E-1
11	7.08425E-3	-2.07291E-1	-9.48995E-1	71	3.53825E-3	-5.66991E-1	-3.96835E-1
12	7.08425E-3	2.07291E-1	-9.48995E-1	72	5.78064E-3	-1.97905E-1	-3.96835E-1
13	0	-3.99349E-1	-9.16799E-1	73	5.78064E-3	1.97905E-1	-3.96835E-1
14	9.01350E-3	-2.62555E-1	-9.16799E-1	74	3.53825E-3	5.66991E-1	-3.96835E-1
15	9.01350E-3	2.62555E-1	-9.16799E-1	75	7.75565E-3	8.59514E-1	-3.96835E-1
16	0	-4.72796E-1	-8.81172E-1	76	0	-9.44812E-1	-3.27613E-1
17	5.63869E-3	-4.11087E-1	-8.81172E-1	77	4.89468E-3	-9.22954E-1	-3.27613E-1
18	3.16131E-3	-1.43488E-1	-8.81172E-1	78	3.86282E-3	-7.65692E-1	-3.27613E-1
19	3.16131E-3	1.43488E-1	-8.81172E-1	79	5.13536E-3	-5.05099E-1	-3.27613E-1
20	5.63869E-3	4.11087E-1	-8.81172E-1	80	3.64389E-3	-1.76303E-1	-3.27613E-1
21	0	-5.37046E-1	-8.43553E-1	81	3.64389E-3	1.76303E-1	-3.27613E-1
22	6.41385E-3	-4.66952E-1	-8.43553E-1	82	5.13536E-3	5.05099E-1	-3.27613E-1
23	3.59590E-3	-1.62988E-1	-8.43553E-1	83	3.86282E-3	7.65692E-1	-3.27613E-1
24	3.59590E-3	1.62988E-1	-8.43553E-1	84	4.89468E-3	9.22954E-1	-3.27613E-1
25	6.41385E-3	4.66952E-1	-8.43553E-1	85	0	-9.66490E-1	-2.56704E-1
26	0	-5.98374E-1	-8.01217E-1	86	5.00102E-3	-9.44130E-1	-2.56704E-1
27	7.14976E-3	-5.20275E-1	-8.01217E-1	87	3.94674E-3	-7.83260E-1	-2.56704E-1
28	4.00849E-3	-1.81600E-1	-8.01217E-1	88	5.24693E-3	-5.16688E-1	-2.56704E-1
29	4.00849E-3	1.81600E-1	-8.01217E-1	89	3.72306E-3	-1.80348E-1	-2.56704E-1
30	7.14976E-3	5.20275E-1	-8.01217E-1	90	3.72306E-3	1.80348E-1	-2.56704E-1
31	0	-6.56401E-1	-7.54412E-1	91	5.24693E-3	5.16688E-1	-2.56704E-1
32	7.84547E-3	-5.70729E-1	-7.54412E-1	92	3.94674E-3	7.83260E-1	-2.56704E-1
33	4.39853E-3	-1.99211E-1	-7.54412E-1	93	5.00102E-3	9.44130E-1	-2.56704E-1
34	4.39853E-3	1.99211E-1	-7.54412E-1	94	0	-9.82847E-1	-1.84425E-1
35	7.84547E-3	5.70729E-1	-7.54412E-1	95	5.08580E-3	-9.60108E-1	-1.84425E-1
36	0	-7.11034E-1	-7.03158E-1	96	4.01365E-3	-7.96516E-1	-1.84425E-1
37	8.57529E-3	-6.18231E-1	-7.03158E-1	97	5.33587E-3	-5.25433E-1	-1.84425E-1
38	4.80771E-3	-2.15791E-1	-7.03158E-1	98	3.78617E-3	-1.83400E-1	-1.84425E-1
39	4.80771E-3	2.15791E-1	-7.03158E-1	99	3.78617E-3	1.83400E-1	-1.84425E-1
40	8.57529E-3	6.18231E-1	-7.03158E-1	100	5.33587E-3	5.25433E-1	-1.84425E-1
41	0	-7.61567E-1	-6.48086E-1	101	4.01365E-3	7.96516E-1	-1.84425E-1
42	6.42875E-3	-7.13133E-1	-6.48086E-1	102	5.08580E-3	9.60108E-1	-1.84425E-1
43	2.93289E-3	-4.70428E-1	-6.48086E-1	103	0	-9.93815E-1	-1.11045E-1
44	4.79164E-3	-1.64201E-1	-6.48086E-1	104	5.15474E-3	-9.70823E-1	-1.11045E-1
45	4.79164E-3	1.64201E-1	-6.48086E-1	105	4.06806E-3	-8.05405E-1	-1.11045E-1
46	2.93289E-3	4.70428E-1	-6.48086E-1	106	5.40820E-3	-5.31297E-1	-1.11045E-1
47	6.42875E-3	7.13133E-1	-6.48086E-1	107	3.83750E-3	-1.85447E-1	-1.11045E-1
48	0	-8.07567E-1	-5.89776E-1	108	3.83750E-3	1.85447E-1	-1.11045E-1
49	6.81415E-3	-7.56207E-1	-5.89776E-1	109	5.40820E-3	5.31297E-1	-1.11045E-1
50	3.10872E-3	-4.98843E-1	-5.89776E-1	110	4.06806E-3	8.05405E-1	-1.11045E-1
51	5.07890E-3	-1.74119E-1	-5.89776E-1	111	5.15474E-3	9.70823E-1	-1.11045E-1
52	5.07890E-3	1.74119E-1	-5.89776E-1	112	0	-9.99313E-1	-3.70540E-2
53	3.10872E-3	4.98843E-1	-5.89776E-1	113	5.17107E-3	-9.76194E-1	-3.70540E-2
54	6.81415E-3	7.56207E-1	-5.89776E-1	114	4.08094E-3	-8.09860E-1	-3.70540E-2
55	0	-8.49108E-1	-5.28222E-1	115	5.42534E-3	-5.34236E-1	-3.70540E-2
56	7.16550E-3	-7.95106E-1	-5.28222E-1	116	3.84965E-3	-1.86473E-1	-3.70540E-2
57	3.26901E-3	-5.24503E-1	-5.28222E-1	117	3.84965E-3	1.86473E-1	-3.70540E-2
58	5.34077E-3	-1.83075E-1	-5.28222E-1	118	5.42534E-3	5.34236E-1	-3.70540E-2
59	5.34077E-3	1.83075E-1	-5.28222E-1	119	4.08094E-3	8.09860E-1	-3.70540E-2
60	3.26901E-3	5.24503E-1	-5.28222E-1	120	5.17107E-3	9.76194E-1	-3.70540E-2

[a]One half of quadrature is shown. The w's, μ's, and η's are the same for the last 120 angles with the signs on the η's being reversed.

[b]Weights associated with each direction of particle travel.

[c]Cosines of the angles between the direction of particle travel and the radius of the cylinder.

[d]Cosines of the angles between the direction of particle travel and the axis of the cylinder.

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