

## Chapter 8 Appendix 2

### SELECTION OF IMPORTANT MONTE CARLO HISTORIES

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The 1986 Dosimetry System (DS86) for Japanese A-bomb survivors uses information describing the behavior of individual radiation particles, simulated by Monte Carlo methods, to calculate the transmission of radiation into structures and, thence, into humans. However, there are practical constraints on the number of such particle "histories" that may be used. First, the number must be sufficiently high to provide adequate statistical precision for any calculated quantity of interest. For integral quantities, such as dose or kerma, statistical precision of approximately 5% (standard deviation) is required to ensure that statistical uncertainties are not a major contributor to the overall uncertainty of the transmitted value. For differential quantities, such as scalar fluence spectra, 10 to 15% standard deviation on individual energy groups is adequate. Second, the number of histories cannot be so large as to require an unacceptably large amount of computer time to process the entire survivor data base. Given that there are approximately 30,000 survivors, each having 13 or 14 organs of interest, the number of histories per organ must be constrained to less than several ten's of thousands at the very most.

Selection and use of only the most important Monte Carlo leakage histories from among all those calculated allows the creation of an efficient house and organ radiation transmission system for use at RERF. While attempts have been made during the adjoint Monte Carlo calculation to bias the histories toward an efficient dose estimate, this effort has been far from satisfactory. Many of the adjoint histories on a typical leakage tape are either starting in an energy group in which there is very little kerma or dose or leaking into an energy group with very little free-field fluence to couple with. By knowing the typical free-field fluence and the fluence-to-dose factors with which the leaking histories will be used, one can select histories from a leakage tape that will contribute to dose and reject the histories which add little dose information. As will be shown, a large number of histories can be rejected without causing an unacceptable increase in the statistical uncertainty of the desired dose calculation.

**Method**

The history selection method can be outlined as follows:

1. Run an adjoint Monte Carlo transport code to produce a leakage tape that has more than enough histories to sift through (e.g., 400,000 to 600,000).
2. Use a forward-adjoint fluence coupling code to couple this leakage tape to a free-field fluence tape. Score the dose according to each starting and leaking energy group combination. Also calculate the variance of each of these doses by analyzing the variation of the dose between the batches of histories.
3. Minimize the Monte Carlo uncertainty on the total dose by selecting a subset of histories such that the number of histories for each combination of starting-leaking energy groups are proportional to  $\text{DOSE}_{ij} * \text{FSD}_{ij} * \sqrt{N_{ij}}$ .

The  $\text{DOSE}_{ij}$  is the dose resulting from free-field fluence group i resulting in dose in kerma group j.  $\text{FSD}_{ij}$  is the Monte Carlo fractional standard deviation of  $\text{DOSE}_{ij}$ . An  $N_{ij}$  is the number of histories needed to get  $\text{FSD}_{ij}$ .

**Mathematical Basis**

The mathematical basis for this procedure is as follows. The total dose from a forward-adjoint coupling calculation is determined by:

$$\begin{aligned}\text{Total dose} &= \sum_j R_j \phi_j \\ &= \sum_j R_j \sum_i \phi_i T_{ij}\end{aligned}\quad (1)$$

where

- $\phi_i$  = free field fluence for energy group i,
- $\phi_j$  = shielded fluence for energy group j,
- $T_{ij}$  = fluence transfer from group i (free field) to group j (shielded), and
- $R_j$  = response function for each energy group j.

If it is assumed that the variance of a transfer matrix element  $T_{ij}$  is independent of other elements, then the total variance of the dose can be calculated by:

$$\begin{aligned}\sigma^2 &= \sum_j R_j^2 \sigma_j^2 \\ &= \sum_j R_j^2 \sum_i (\phi_i \sigma_{ij})^2 \\ &= \sum_j R_j^2 \sum_i (\phi_i T_{ij} FSC_{ij})^2\end{aligned}\quad (2)$$

where

- $\sigma^2$  = the variance of the total dose,
- $\sigma_j^2$  = the variance of shielded fluence group j,
- $\sigma_{ij}^2$  = the variance of the individual transfer matrix element  $T_{ij}$ , and
- $FSC_{ij}$  = the FSD of transfer matrix element  $T_{ij}$   
(it is the common output statistic of the DRC coupling system).

Finally, it is assumed that the  $FSD_{ij}$  is proportional, with constants of proportionality  $K_{ij}$ , to the inverse square root of the number of histories  $N_{ij}$  on the adjoint leakage tape going from group i to group j,

$$FSD_{ij} = \frac{K_{ij}}{\sqrt{N_{ij}}} \quad (3)$$

While the assumption of independence may not be entirely accurate, the results of our assumption can be tested later with real calculations.

Minimization of  $\sigma^2$  with respect to the number of histories going from k to l,  $N_{kl}$ , will provide an optimum number of histories in each element of the transfer matrix. This is accomplished as follows:

$$\begin{aligned} \frac{\delta\sigma^2}{\delta N_{kl}} &= 0 \\ &= \sum_j R_j^2 \frac{\delta\sigma^2}{\delta N_{kl}} = \sum_j R_j^2 \sum_i \phi_i^2 T_{ij}^2 \frac{\delta FSD_{ij}^2}{\delta N_{kl}} \end{aligned}$$

From Equation (3)

$$0 = \sum_j R_j^2 \sum_i \phi_i^2 T_{ij}^2 K_{ij}^2 \frac{\sigma[\frac{1}{N_{ij}}]}{N_{kl}} = - \sum_j R_j^2 \sum_i \frac{\sigma_i^2 T_{ij}^2 K_{ij}^2}{N_{ij}^2} \frac{\sigma N_{ij}}{\sigma N_{kl}} \quad (4)$$

Now evaluate:  $\delta N_{ij}/\delta N_{kl}$ .

$$\frac{\delta N_{ij}}{\delta N_{kl}} = 1, \text{ if } i = k \text{ and } j = l,$$

but

$$\frac{\delta N_{ij}}{\delta N_{kl}} = -C_{ijkl}, \text{ if } i \neq k \text{ or } j \neq l.$$

Also

$$N = \sum_j \sum_i N_{ij} \quad (5)$$

$$\frac{\delta N}{\delta N_{kl}} = 0 = \sum_j \sum_i \frac{\delta N_{ij}}{\delta N_{kl}} = \left[ \sum_{j \neq i} \sum_{i \neq k} \frac{\delta N_{ij}}{\delta N_{kl}} \right] + \frac{\delta N_{ij}}{\delta N_{ij}}$$

$$0 = \left[ \sum_{j \neq l} \sum_{i \neq k} -C_{ijkl} \right] + 1 \quad (6)$$

Thus, on returning to the minimization of  $\sigma^2$ ,

$$\frac{R_1^2 \phi_k^2 T_{kl}^2 K_{kl}^2}{N_{kl}^2} = \sum_{j \neq l} R_j^2 \sum_{i \neq l} \frac{\delta_i^2 T_{ij}^2 K_{ij}^2 C_{ijkl}^2}{N_{ij}^2}$$

This is solved when

$$N_{kl}^2 = R_1^2 \phi_k^2 T_{kl}^2 K_{kl}^2 A \quad (7)$$

where  $A$  is a proportionality constant that can be used for normalizing the total number of histories. To determine the constant  $K_{kl}$ , a coupling calculation can be used to which calculates  $FSD_{kl}^o$  from a number of histories  $N_{kl}^o$ . Then:

$$K_{kl}^2 = FSD_{kl}^{o2} N_{kl}^o \quad (8)$$

$$N_{kl}^2 = R_1^2 \phi_k^2 T_{kl}^2 FSD_{kl}^{o2} N_{kl}^o A \quad (9)$$

$A$  is determined from Equation (5)

$$N = A \sum_1 R_1 \sum_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o} \quad (10)$$

and

$$A = \frac{N}{\sum_1 R_1 \sum_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}} \quad (11)$$

Thus, the optimal number  $N_{kl}$  is found to be:

$$N_{kl} = N \frac{R_1 \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}}{\sum_1 R_1 \sum_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}} \quad (12)$$

This expression can be evaluated in a two-step process to separate the process of free-field fluence optimization from that of dose-response optimization. Rewriting the expression in two parts is done as follows:

$$N_{kl} = N_1 \frac{\phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}}{\sum_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}} \quad (13)$$

$$N_1 = N \frac{R_1 \sum_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}}{\sum_1 R_1 \sum_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}} \quad (14)$$

The first expression selects the number of histories leaking in each free-field group  $k$  given a shielded group 1. The second expression tells how to optimize the various shielded groups. The operation could be performed also in one step if desired.

The two equations above have also been modified so that emphasis can be put on part of the free-field spectrum or on part of the response function.

$$N_{kl} = N_1 \frac{F_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}}{\sum_k F_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}} \quad (15)$$

$$N_1 = N \frac{M_1 R_1 \sum_k F_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}}{\sum_1 M_1 R_1 \sum_k F_k \phi_k T_{kl} FSD_{kl}^o \sqrt{N_{kl}^o}} \quad (16)$$

were  $F_k$  is a fluence multiplier for group k and  $M_1$  is a response multiplier for group 1. This modification is necessary so that in situations where certain parts of the spectrum (e.g., neutrons) do not contribute much dose, their contribution to improve their statistics can be arbitrarily increased.

Another practical concern is to ensure that  $N_{kl}$ , the number of desired histories, is not larger than  $N_{kl}^o$ , the number of available histories on the large leakage tape. Also, it might be wise not to reject too many of the unimportant histories because as the number of histories in any transfer matrix element is reduced the chance of a Monte Carlo anomaly increases. Then the hoped for improvement in statistics may fail to appear.

### Example

The optimization procedure was carried out for a phantom organ adjoint data set placed in the Hiroshima free field. The result illustrates the use and benefit of this method for reducing coupling time.

First, an adjoint MORSE-VCS calculation was performed to produce a large-number-of-histories leakage tape. The specific details are as follows:

Phantom: Japanese adult

Organ: marrow

Starting histories: 400,000

Leaking histories: 582,233

Orientation: left side to burst, standing.

Second, a tape of free-field fluence was prepared. The specific details are as follows:

City: Hiroshima

Ground range: 1500 m

Radiation: neutrons, prompt and secondary gamma rays.

The phantom shielding and free-field fluences were coupled using the DRC code that is part of the VCS system. Modifications were made to calculate also the fluence transfer matrix,  $T_{kl}$ ; the number matrix,  $N_{kl}^o$ ; and the FSD matrix,  $FSD_{kl}^o$ , in addition to the usual DRC output. Table 1 shows the main radiation doses and the FSD associated with them. Tables 2 to 4 show the  $T_{kl}$ ,  $N_{kl}^o$ , and  $FSD_{kl}^o$  obtained from this coupling that used all the histories.

Based on the experience at RERF this coupling would cost approximately two minutes of CPU time and eight minutes of clock time per individual organ. For 10 organs and 10,000 survivors that would be about 5 months of CPU time and 18 months of clock time. Such long times are unacceptable for practice.

## SELECTION OF MONTE CARLO HISTORIES

Table 1. Comparison of Organ Doses and FSDs with Various History Biasing Schemes.  
Hiroshima, 1500 m Ground Range, Marrow ( $\approx 600,000$  Leaking Histories)

	99% Reduction in Histories						
	All Histories	Random Rejection	Leaking Bias	Starting Bias Also	Leaking* Bias	Starting* Bias Also	Dec 84 System
DOSES (rad)							
Neutron	.261	.241	.265	.269	.259	.259	.274
Prompt Gamma	19.104	20.460	16.762	18.553	18.559	18.918	18.859
Debris Gamma	23.681	24.495	23.934	21.844	21.526	22.864	25.101
Body Gamma	.198	.189	.262	.199	.180	.205	.157
FSDs (in percent)							
Neutron	1.0(9.9)	10.4(9.4)	5.9(5.9)	24.3(27.5)	5.7(5.9)	4.8(4.9)	4.9
Prompt Gamma	1.4(1.5)	13.3(14.8)	9.2(11.1)	4.4(4.6)	11.6(11.3)	5.3(5.7)	5.8
Debris Gamma	1.5	13.9	8.7	6.01	10.1	7.7	6.1
Body Gamma	2.1(1.2)	9.8(11.9)	21.4(29.9)	31.1(30.2)	7.5(7.1)	5.3(5.3)	8.1
# of Histories	582233	5822	5822	5822	5822	5822	29549
Estimated Clock Time for 10 organs 10,000 survivors	18 month	6 days	6 days	6 days	6 days	6 days	30 days

\*Neutron fluence multiplied by 50.0 to improve statistics on neutron components.

( ) values in parenthesis indicate estimated values by summing individual variances.

A prediction of the total FSDs on the major dose components was made by summing the variances in Tables 2 to 4. This procedure assumes a reasonable accuracy in the FSDs. Table 1 indicates the predicted FSD results in parentheses. The comparison with actual FSDs is good except for body gamma. It is likely that the exception is caused by the large FSDs in the transfer matrix of this dose component.

Next, the results of several history biasing methods were illustrated for a 99% reduction in the number of histories. This smaller number of histories reduces the estimated clock time to about four seconds per individual per organ. It would reduce the total time to do 10 organs for 10,000 survivors to six days.

Random rejection of histories to leave only 5,822 of them results in doses and FSDs as shown in Table 1. The new matrices  $N_{kl}$  and  $FSD_{kl}$  are shown in Tables 5 and 6.

Next, histories were selected based on the importance of their leaking energy. The doses and FSDs are shown in Table 1. The new matrices for  $N_{kl}$  and  $FSD_{kl}$  are shown in Tables 7 and 8. Note the different emphasis on the number of histories leaking out at large neutron energies. Because there is little free-field fluence at the high energies, very few histories are selected that leak at high energies.

Finally, histories were chosen based on the importance of their leaking energy and on their starting energy. The major doses and FSDs are again shown in Table 1. The new matrices  $N_{kl}$  and  $FSD_{kl}$  are shown in Tables 9 and 10. Note that this time there are very few neutrons histories at all. This is because the neutron dose is only a small part of the total dose. The loss of histories is unacceptable because, while the neutrons are relatively unimportant in the total kerma or dose, they are important because they produce more effect per unit dose. Thus, the histories were reselected based on the importance of their leaking

Table 2. Fluence Transfer Matrix  $\phi_k T_{kl}$  for Hiroshima 1500 m Ground Range. Fluence Coupled with Marrow Leakage Tape (582,223 Histories)

**Table 2.** Continued

**Table 3. Number Transfer Matrix,  $N_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Marrow Leakage Tape (582,223 Histories)**

**Table 4. FSD Transfer Matrix,  $FSD_{k1}$ , for Hiroshima, 1500 m Ground Range, Coupled with Marrow Leakage Tape (582,223 Histories)**

**Table 5. Number Transfer Matrix,  $N_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Randomly Reduced Marrow Leakage Tape (5,822 Histories)**

UNIFORMLY REDUCED NUMBER OF HISTORIES	NUMBER OF HISTORIES	LEAKING ENERGY GROUP	10	20	30	40	50	55
1	1							
2	2							
3	3							
4	4							
5	5							
6	6							
7	7							
8	8							
9	9							
10	10							
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195	195							
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Table 6. FSD Transfer Matrix,  $FSD_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Randomly Reduced Marrow Leakage Tape (5,822) Histories

		LEAKING ENERGY GROUP	10	20	30	40	50	50
1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81
82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108
109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126
127	128	129	130	131	132	133	134	135
136	137	138	139	140	141	142	143	144
145	146	147	148	149	150	151	152	153
154	155	156	157	158	159	160	161	162
163	164	165	166	167	168	169	170	171
172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198
199	200	201	202	203	204	205	206	207
208	209	210	211	212	213	214	215	216
217	218	219	220	221	222	223	224	225
226	227	228	229	230	231	232	233	234
235	236	237	238	239	240	241	242	243
244	245	246	247	248	249	250	251	252
253	254	255	256	257	258	259	260	261
262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279
280	281	282	283	284	285	286	287	288
289	290	291	292	293	294	295	296	297
298	299	300	301	302	303	304	305	306
307	308	309	310	311	312	313	314	315
316	317	318	319	320	321	322	323	324
325	326	327	328	329	330	331	332	333
334	335	336	337	338	339	340	341	342
343	344	345	346	347	348	349	350	351
352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369
370	371	372	373	374	375	376	377	378
379	380	381	382	383	384	385	386	387
388	389	390	391	392	393	394	395	396
397	398	399	400	401	402	403	404	405
406	407	408	409	410	411	412	413	414
415	416	417	418	419	420	421	422	423
424	425	426	427	428	429	430	431	432
433	434	435	436	437	438	439	440	441
442	443	444	445	446	447	448	449	450
451	452	453	454	455	456	457	458	459
460	461	462	463	464	465	466	467	468
469	470	471	472	473	474	475	476	477
478	479	480	481	482	483	484	485	486
487	488	489	490	491	492	493	494	495
496	497	498	499	500	501	502	503	504
505	506	507	508	509	510	511	512	513
514	515	516	517	518	519	520	521	522
523	524	525	526	527	528	529	530	531
532	533	534	535	536	537	538	539	540
541	542	543	544	545	546	547	548	549
550	551	552	553	554	555	556	557	558
559	560	561	562	563	564	565	566	567
568	569	570	571	572	573	574	575	576
577	578	579	580	581	582	583	584	585
586	587	588	589	590	591	592	593	594
595	596	597	598	599	600	601	602	603
604	605	606	607	608	609	610	611	612
613	614	615	616	617	618	619	620	621
622	623	624	625	626	627	628	629	630
631	632	633	634	635	636	637	638	639
640	641	642	643	644	645	646	647	648
649	650	651	652	653	654	655	656	657
658	659	660	661	662	663	664	665	666
667	668	669	670	671	672	673	674	675
676	677	678	679	680	681	682	683	684
685	686	687	688	689	690	691	692	693
694	695	696	697	698	699	700	701	702
703	704	705	706	707	708	709	710	711
712	713	714	715	716	717	718	719	720
721	722	723	724	725	726	727	728	729
730	731	732	733	734	735	736	737	738
739	740	741	742	743	744	745	746	747
748	749	750	751	752	753	754	755	756
757	758	759	760	761	762	763	764	765
766	767	768	769	770	771	772	773	774
775	776	777	778	779	780	781	782	783
784	785	786	787	788	789	790	791	792
793	794	795	796	797	798	799	800	801
802	803	804	805	806	807	808	809	810
811	812	813	814	815	816	817	818	819
820	821	822	823	824	825	826	827	828
829	830	831	832	833	834	835	836	837
838	839	840	841	842	843	844	845	846
847	848	849	850	851	852	853	854	855
856	857	858	859	860	861	862	863	864
865	866	867	868	869	870	871	872	873
874	875	876	877	878	879	880	881	882
883	884	885	886	887	888	889	890	891
892	893	894	895	896	897	898	899	900
901	902	903	904	905	906	907	908	909
910	911	912	913	914	915	916	917	918
919	920	921	922	923	924	925	926	927
928	929	930	931	932	933	934	935	936
937	938	939	940	941	942	943	944	945
946	947	948	949	950	951	952	953	954
955	956	957	958	959	960	961	962	963
964	965	966	967	968	969	970	971	972
973	974	975	976	977	978	979	980	981
982	983	984	985	986	987	988	989	990
991	992	993	994	995	996	997	998	999
999	999	999	999	999	999	999	999	999

**Table 7. Number Transfer Matrix,  $N_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Optimized Leaking History Marrow Leakage Tape (5,822 Histories)**

Table 8. FSD Transfer Matrix,  $FSD_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Optimized Leaking History Marrow Leakage Tape (5,822 Histories)

Hist	LEAKING ENERGY GROUP	10										20										30										50										60										70										80										90										100																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282</th

Table 9. Number Transfer Matrix,  $N_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Optimized Leaking and Starting History Marrow Leakage Tape (5,822 Histories)

STARTING HISTORIES	OPTIMIZED RESULTS DISPLAYED	LEAKING ENERGY GROUP	20	30	40	50	50
1	1						
2	2						
3	3						
4	4						
5	5						
6	6						
7	7						
8	8						
9	9						
10	10						
11	11						
12	12						
13	13						
14	14						
15	15						
16	16						
17	17						
18	18						
19	19						
20	20						
21	21						
22	22						
23	23						
24	24						
25	25						
26	26						
27	27						
28	28						
29	29						
30	30						
31	31						
32	32						
33	33						
34	34						
35	35						
36	36						
37	37						
38	38						
39	39						
40	40						
41	41						
42	42						
43	43						
44	44						
45	45						
46	46						
47	47						
48	48						
49	49						
50	50						
51	51						
52	52						
53	53						
54	54						
55	55						
56	56						
57	57						
58	58						
59	59						
60	60						
61	61						
62	62						
63	63						
64	64						
65	65						
66	66						
67	67						
68	68						
69	69						
70	70						
71	71						
72	72						
73	73						
74	74						
75	75						
76	76						
77	77						
78	78						
79	79						
80	80						
81	81						
82	82						
83	83						
84	84						
85	85						
86	86						
87	87						
88	88						
89	89						
90	90						
91	91						
92	92						
93	93						
94	94						
95	95						
96	96						
97	97						
98	98						
99	99						
100	100						
101	101						
102	102						
103	103						
104	104						
105	105						
106	106						
107	107						
108	108						
109	109						
110	110						
111	111						
112	112						
113	113						
114	114						
115	115						
116	116						
117	117						
118	118						
119	119						
120	120						
121	121						
122	122						
123	123						
124	124						
125	125						
126	126						
127	127						
128	128						
129	129						
130	130						
131	131						
132	132						
133	133						
134	134						
135	135						
136	136						
137	137						
138	138						
139	139						
140	140						
141	141						
142	142						
143	143						
144	144						
145	145						
146	146						
147	147						
148	148						
149	149						
150	150						
151	151						
152	152						
153	153						
154	154						
155	155						
156	156						
157	157						
158	158						
159	159						
160	160						
161	161						
162	162						
163	163						
164	164						
165	165						
166	166						
167	167						
168	168						
169	169						
170	170						
171	171						
172	172						
173	173						
174	174						
175	175						
176	176						
177	177						
178	178						
179	179						
180	180						
181	181						
182	182						
183	183						
184	184						
185	185						
186	186						
187	187						
188	188						
189	189						
190	190						
191	191						
192	192						
193	193						
194	194						
195	195						
196	196						
197	197						
198	198						
199	199						
200	200						
201	201						
202	202						
203	203						
204	204						
205	205						
206	206						
207	207						
208	208						
209	209						
210	210						
211	211						
212	212						
213	213						
214	214			</			

Table 10. FSD Transfer Matrix,  $FSD_{k1}$ , for Hiroshima, 1500 m Ground Range. Fluence Coupled with Optimized Leaking and Starting History Marrow Leakage Tape (5,822 Histories)

Table 11. Number Transfer Matrix,  $N_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence (Neutrons Increased 50 Times) Coupled with Optimized Leaking History Marrow Leakage Tape (5,822 Histories)

LEAKING HISTORIES OPTIMIZED RESULTS DISPLAYED NUMBER OF HISTORIES	LEAKING ENERGY GROUP									
	20	30	40	50	60	70	80	90	100	110
57	1	2	3	4	5	6	7	8	9	10
58	1	2	3	4	5	6	7	8	9	10
59	1	2	3	4	5	6	7	8	9	10
60	1	2	3	4	5	6	7	8	9	10
61	1	2	3	4	5	6	7	8	9	10
62	1	2	3	4	5	6	7	8	9	10
63	1	2	3	4	5	6	7	8	9	10
64	1	2	3	4	5	6	7	8	9	10
65	1	2	3	4	5	6	7	8	9	10
66	1	2	3	4	5	6	7	8	9	10
67	1	2	3	4	5	6	7	8	9	10
68	1	2	3	4	5	6	7	8	9	10
69	1	2	3	4	5	6	7	8	9	10
70	1	2	3	4	5	6	7	8	9	10
71	1	2	3	4	5	6	7	8	9	10
72	1	2	3	4	5	6	7	8	9	10
73	1	2	3	4	5	6	7	8	9	10
74	1	2	3	4	5	6	7	8	9	10
75	1	2	3	4	5	6	7	8	9	10
76	1	2	3	4	5	6	7	8	9	10
77	1	2	3	4	5	6	7	8	9	10
78	1	2	3	4	5	6	7	8	9	10
79	1	2	3	4	5	6	7	8	9	10
80	1	2	3	4	5	6	7	8	9	10
81	1	2	3	4	5	6	7	8	9	10
82	1	2	3	4	5	6	7	8	9	10
83	1	2	3	4	5	6	7	8	9	10
84	1	2	3	4	5	6	7	8	9	10
85	1	2	3	4	5	6	7	8	9	10
86	1	2	3	4	5	6	7	8	9	10
87	1	2	3	4	5	6	7	8	9	10
88	1	2	3	4	5	6	7	8	9	10
89	1	2	3	4	5	6	7	8	9	10
90	1	2	3	4	5	6	7	8	9	10
91	1	2	3	4	5	6	7	8	9	10
92	1	2	3	4	5	6	7	8	9	10
93	1	2	3	4	5	6	7	8	9	10
94	1	2	3	4	5	6	7	8	9	10
95	1	2	3	4	5	6	7	8	9	10
96	1	2	3	4	5	6	7	8	9	10
97	1	2	3	4	5	6	7	8	9	10
98	1	2	3	4	5	6	7	8	9	10
99	1	2	3	4	5	6	7	8	9	10
100	1	2	3	4	5	6	7	8	9	10
101	1	2	3	4	5	6	7	8	9	10
102	1	2	3	4	5	6	7	8	9	10
103	1	2	3	4	5	6	7	8	9	10
104	1	2	3	4	5	6	7	8	9	10
105	1	2	3	4	5	6	7	8	9	10
106	1	2	3	4	5	6	7	8	9	10
107	1	2	3	4	5	6	7	8	9	10
108	1	2	3	4	5	6	7	8	9	10
109	1	2	3	4	5	6	7	8	9	10
110	1	2	3	4	5	6	7	8	9	10

**Table 12. FSD Transfer Matrix,  $FSD_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence (Neutrons Increased 50 Times) Coupled with Optimized Leaking History Marrow Leakage Tape (5,822 Histories)**

Table 13. Transfer Matrix,  $N_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence (Neutrons Increased 50 Times)  
Coupled with Optimized Leaking and Starting History Marrow Leakage Tape (5,822 Histories)

STARTING HISTORIES NUMBER OF HISTORIES	OPTIMIZED RESULTS DISPLAYED	LEAKING ENERGY GROUP	20		30		40		50	
			1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9
10	10	10	10	10	10	10	10	10	10	10
11	11	11	11	11	11	11	11	11	11	11
12	12	12	12	12	12	12	12	12	12	12
13	13	13	13	13	13	13	13	13	13	13
14	14	14	14	14	14	14	14	14	14	14
15	15	15	15	15	15	15	15	15	15	15
16	16	16	16	16	16	16	16	16	16	16
17	17	17	17	17	17	17	17	17	17	17
18	18	18	18	18	18	18	18	18	18	18
19	19	19	19	19	19	19	19	19	19	19
20	20	20	20	20	20	20	20	20	20	20
21	21	21	21	21	21	21	21	21	21	21
22	22	22	22	22	22	22	22	22	22	22
23	23	23	23	23	23	23	23	23	23	23
24	24	24	24	24	24	24	24	24	24	24
25	25	25	25	25	25	25	25	25	25	25
26	26	26	26	26	26	26	26	26	26	26
27	27	27	27	27	27	27	27	27	27	27
28	28	28	28	28	28	28	28	28	28	28
29	29	29	29	29	29	29	29	29	29	29
30	30	30	30	30	30	30	30	30	30	30
31	31	31	31	31	31	31	31	31	31	31
32	32	32	32	32	32	32	32	32	32	32
33	33	33	33	33	33	33	33	33	33	33
34	34	34	34	34	34	34	34	34	34	34
35	35	35	35	35	35	35	35	35	35	35
36	36	36	36	36	36	36	36	36	36	36
37	37	37	37	37	37	37	37	37	37	37
38	38	38	38	38	38	38	38	38	38	38
39	39	39	39	39	39	39	39	39	39	39
40	40	40	40	40	40	40	40	40	40	40
41	41	41	41	41	41	41	41	41	41	41
42	42	42	42	42	42	42	42	42	42	42
43	43	43	43	43	43	43	43	43	43	43
44	44	44	44	44	44	44	44	44	44	44
45	45	45	45	45	45	45	45	45	45	45
46	46	46	46	46	46	46	46	46	46	46
47	47	47	47	47	47	47	47	47	47	47
48	48	48	48	48	48	48	48	48	48	48
49	49	49	49	49	49	49	49	49	49	49
50	50	50	50	50	50	50	50	50	50	50
51	51	51	51	51	51	51	51	51	51	51
52	52	52	52	52	52	52	52	52	52	52
53	53	53	53	53	53	53	53	53	53	53
54	54	54	54	54	54	54	54	54	54	54
55	55	55	55	55	55	55	55	55	55	55
56	56	56	56	56	56	56	56	56	56	56
57	57	57	57	57	57	57	57	57	57	57
58	58	58	58	58	58	58	58	58	58	58
59	59	59	59	59	59	59	59	59	59	59
60	60	60	60	60	60	60	60	60	60	60
61	61	61	61	61	61	61	61	61	61	61
62	62	62	62	62	62	62	62	62	62	62
63	63	63	63	63	63	63	63	63	63	63
64	64	64	64	64	64	64	64	64	64	64
65	65	65	65	65	65	65	65	65	65	65
66	66	66	66	66	66	66	66	66	66	66
67	67	67	67	67	67	67	67	67	67	67
68	68	68	68	68	68	68	68	68	68	68
69	69	69	69	69	69	69	69	69	69	69
70	70	70	70	70	70	70	70	70	70	70
71	71	71	71	71	71	71	71	71	71	71
72	72	72	72	72	72	72	72	72	72	72
73	73	73	73	73	73	73	73	73	73	73
74	74	74	74	74	74	74	74	74	74	74
75	75	75	75	75	75	75	75	75	75	75
76	76	76	76	76	76	76	76	76	76	76
77	77	77	77	77	77	77	77	77	77	77
78	78	78	78	78	78	78	78	78	78	78
79	79	79	79	79	79	79	79	79	79	79
80	80	80	80	80	80	80	80	80	80	80
81	81	81	81	81	81	81	81	81	81	81
82	82	82	82	82	82	82	82	82	82	82
83	83	83	83	83	83	83	83	83	83	83
84	84	84	84	84	84	84	84	84	84	84
85	85	85	85	85	85	85	85	85	85	85
86	86	86	86	86	86	86	86	86	86	86
87	87	87	87	87	87	87	87	87	87	87
88	88	88	88	88	88	88	88	88	88	88
89	89	89	89	89	89	89	89	89	89	89
90	90	90	90	90	90	90	90	90	90	90
91	91	91	91	91	91	91	91	91	91	91
92	92	92	92	92	92	92	92	92	92	92
93	93	93	93	93	93	93	93	93	93	93
94	94	94	94	94	94	94	94	94	94	94
95	95	95	95	95	95	95	95	95	95	95
96	96	96	96	96	96	96	96	96	96	96
97	97	97	97	97	97	97	97	97	97	97
98	98	98	98	98	98	98	98	98	98	98
99	99	99	99	99	99	99	99	99	99	99
100	100	100	100	100	100	100	100	100	100	100

Table 14. FSD Transfer Matrix,  $FSD_{kl}$ , for Hiroshima, 1500 m Ground Range. Fluence (Neutrons Increased 50 Times) Coupled with Optimized Leaking and Starting History Marrow Leakage Tape (5,822 Histories)

hist.	LEAKING ENERGY GROUP	10					20					30					40					50					
		10	20	30	40	50	10	20	30	40	50	10	20	30	40	50	10	20	30	40	50	10	20	30	40	50	
1	296	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	429	244	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	154711239	382	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	25492016151582	534	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	17911691173471724	299	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	229624742446255711841	774	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	289416532118254116672272	328	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	2248192422743236113393145	931	266	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	28142315205432414562343233	951	176	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	2711986122344451	448276933884	939	582	98	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	245116772562320791	1189154681275	544	349	91	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	2346229122282319168212289811168	519	315	243	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1476142038857519243824114681476	573	315	251	185	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	3443795446578519243824114681476	573	315	251	185	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	24442366298488616729472	7481515	473	446	349	211	93	35	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	184519915052233	4262396251481947	764	349	131	72	69	52	21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	2297192456929413788545627481343	425	912	399	294	191	117	72	51	22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
18	6717635470287755718	448276933884	1271872	646	274	248	123	128	8	8	8	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
19	229278722776159244566222422893	616	553	287	122	145	142	73	51	34	21	8	4	1	1	1	1	1	1	1	1	1	1	1	1	1	
20	19552475273	30991832249951668	829	644	445	189	142	94	73	59	34	25	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	267427944624591222518427812581433	885	477	238	189	134	83	61	118	41	26	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
22	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
23	724466448853493121289831292	864	448	251	294	188	93	47	45	23	14	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
24	24495853493121289831292	864	448	251	294	188	93	47	45	23	14	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
25	22985853493121289831292	864	448	251	294	188	93	47	45	23	14	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
26	22985853493121289831292	864	448	251	294	188	93	47	45	23	14	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
27	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
28	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
29	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
30	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
31	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
32	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
33	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
34	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
35	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
36	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
37	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
38	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
39	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
40	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
41	222525734261348122895477	72486251	784	585	342	131	165	124	82	55	35	21	16	1	1	1	1	1	1	1	1	1	1	1	1	1	
42	222525734261348122895477	72486251	784	585																							

energy with the neutrons increased arbitrarily by 50 times. Table 1 shows the major doses and FSDs. Tables 11 and 12 show the matrices  $N_{kl}$  and  $FSD_{kl}$ .

Finally, histories were chosen based on both the importance of their leaking and starting energies, still with the neutron importance increased by 50. Table 1 shows the major doses and FSDs. Tables 13 and 14 show the matrices  $N_{kl}$  and  $FSD_{kl}$ . This is the method of biasing that we recommend for calculating the dose in the phantom.

The final column of Table 1 gives the results from the RERF coupling system used in December 1984, which used unbiased leakage data sets of approximately 30,000 histories each. The doses are about the same in spite of minor phantom and organ distribution changes. The FSDs are comparable to those for the recommended biasing method and the clock time for the recommended biasing method is about one fifth that experienced using the unbiased system.

### Concluding Comments

By preprocessing the leakage tape the important histories can be saved and used in calculating dose. Some information is sacrificed of course. Energy groups with small doses will have very large FSDs. The tapes optimized for calculating dose are thus unusable for calculating spectra. If spectra are desired a similar optimization procedure for dose in each energy group is available. For example, roughly 40,000 histories are required to get a 10% FSD in each of the DLC-31 groups.

As the number of histories used is reduced, the Monte Carlo FSDs increase. Figures 1 and 2 show the result for reducing the number of histories from a typical organ calculation. In Figure 1 the FSDs for each dose component are plotted as a function of the number of histories making up that dose component. The number of neutron and neutron-induced gamma-ray histories can be reduced by a factor of 10 or the gamma-ray histories reduced by a factor of 5 before dose-important histories are removed. Thereafter the FSD will be inversely proportional to the square root of the number of histories and will, therefore, increase as the number of histories is reduced. In the system, the neutron and gamma-ray histories compete for importance. Figure 2 shows how the dose FSD is changed as the total number of all histories is reduced. (The neutron's dose importance has been set at 20 times that of the gamma rays.) A reduction to 1/3 of the number of histories makes no difference in the FSD. A further reduction of 1/20 realigns the dose components according to their importance to dose. Any further reduction just causes the same increase in FSDs as stated above.

For the purposes of creating data sets for DS86, history biasing was used based on the Hiroshima spectrum at 1500 m ground range. To get adequate neutron statistics the neutron fluence was increased by a factor of 20 for the house data sets and 20 for the organ tape. The house histories were biased only for the leaking energies with the free field because this provided a respectable spectrum in every energy group. For each organ, two data sets were produced. One was biased for dose only. The other was biased for spectra in all energy groups. The house data sets consist of about 40,000 histories each. The organ data sets consist of about 6,000 histories per organ for dose and 40,000 histories per organ for spectra. The user may select the data appropriate to his needs. It must be pointed out that the doses calculated from the two organ tapes will be similar but not exactly the same because of the different histories used on the two tapes.

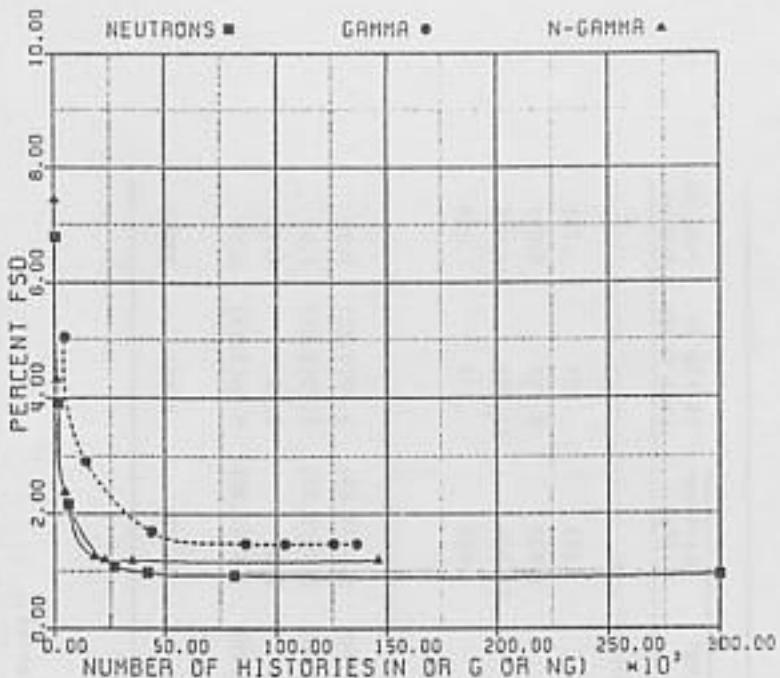


Figure 1. Percent fractional standard deviation as a function of history reduction. (N or G or NG)

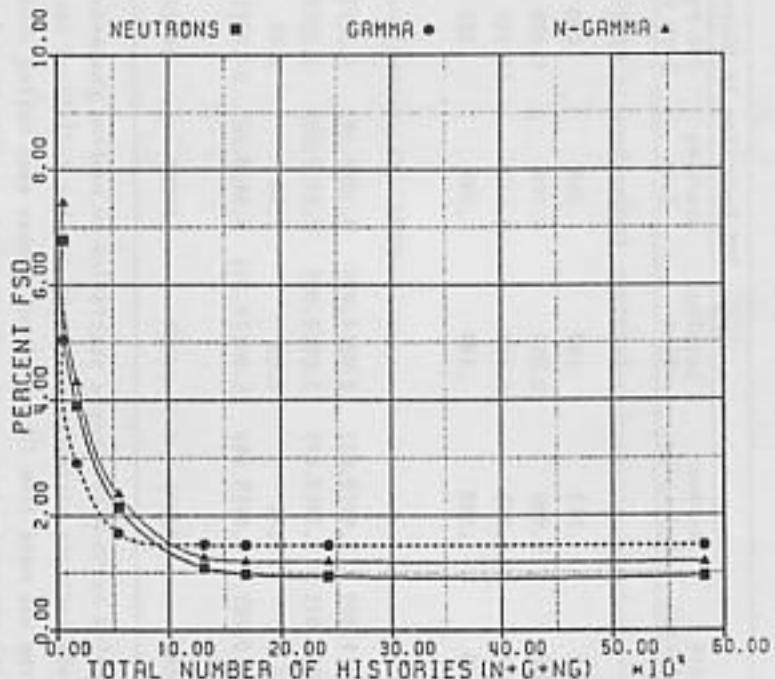


Figure 2. Percent fractional standard deviation as a function of history reduction. (N+G+NG)

The applicability of the optimization procedure at 1500 m to a different orientation, range, or city was investigated. Similar improvements were noted no matter where or how the phantom was coupled. The spectra or organ responses do not change in an amount significant enough to invalidate the optimization procedure.

Similar improvement in the dosimetry system occurs when the house leakage data sets are optimized. Many of the calculated leaking histories do not contribute much fluence in the house. Optimizing the house data to give a good spectrum at a reasonable cost in running

Table 15. Comparison of House Doses and FSDs with Various History Biasing Schemes, Hiroshima,  
1500 m Ground Range. House Type 33. ( $\approx 77,000$  Leaking Histories)

	70% Reduction in Histories								
	All Histories	Random Rejection	Leaking Bias	Starting Bias	10% FSD in Each Group	Leaking* Bias	Starting* Bias	5% FSD in Each Group	June 84 System
-Exposure (House Dose in Rad)-									
Neutron	.271	.271	.269	.259	.248	.277	.271	.274	.261
Prompt Gamma	9.807	9.500	9.854	9.740	9.706	10.044	9.833	9.784	9.054
Delayed Gamma	12.177	11.599	12.029	12.177	11.755	12.773	12.275	12.155	14.085
House Gamma	.178	.180	.146	.193	.165	.177	.179	.178	.168
-FSDs (In percent)-									
Neutron	1.58(1.44)	2.69(2.63)	2.02(1.99)	6.33(6.26)	3.31(3.72)	2.31(1.99)	1.70(1.63)	1.72(1.65)	2.90
Prompt Gamma	2.19(2.01)	3.74(3.67)	3.09(2.90)	2.22(2.08)	3.20(2.03)	3.37(3.40)	2.73(2.33)	2.25(2.05)	4.09
Delayed Gamma	2.13	3.76	2.85	2.33	3.74	3.63	3.08	2.18	4.16
House Gamma	3.65(3.26)	6.62(5.95)	15.86(13.31)	7.50(8.20)	17.10(16.16)	3.80(3.40)	3.67(3.30)	3.65(3.26)	6.04
# of Histories	76750	23025	22000	14009	14927	21975	17179	44889	23000

\*Neutron Fluence multiplied by 50.0 to improve statistics on neutron components.

( ) values in parenthesis indicate estimated values by summing individual variances.

Note: House leakage tape was made from 10 individual houses each having about 25,000 histories.

Combining gives about 250,000 histories. Angle biasing reduces this to about 80,000 histories. Part of the FSD is due to the variation between individual houses ±IX.

Table 16. Comparison of Organ Doses and FSDs with Various House History Biasing Schemes, Hiroshima,  
1500 m Ground Range. House Type 33. Marrow ( $\approx 600,000$  Leaking Histories)

	All Histories	Random Rejection	Leaking Bias	Starting Bias	10% FSD in Each Group	Leaking* Bias	Starting* Bias	5% FSD in Each Group
<b>Marrow Doses</b>								
Neutron	.094(.093)	.092(.084)	.092(.087)	.077(.060)	.089(.081)	.092(.087)	.093(.092)	.094(.092)
Prompt Gamma	7.576(7.282)	7.877(7.551)	7.509(7.276)	7.602(7.271)	7.507(6.928)	7.707(7.309)	7.507(7.066)	7.590(7.285)
Delayed Gamma	8.819(8.910)	8.956(8.357)	8.736(8.669)	8.936(9.011)	9.117(8.416)	9.147(8.460)	8.999(8.647)	8.921(9.006)
House Gamma	.134(.140)	.135(.136)	.120(.106)	.141(.138)	.118(.109)	.132(.136)	.133(.140)	.134(.140)
Body Gamma	.115(.090)	.118(.086)	.116(.087)	.086(.030)	.117(.076)	.116(.087)	.107(.065)	.121(.087)
<b>FSDs (in percent)</b>								
Neutron	.97(4.05)	1.25(4.71)	1.06(3.75)	2.93(15.23)	1.43(6.47)	1.06(3.75)	1.28(4.43)	1.03(4.17)
Prompt Gamma	1.38(6.05)	1.64(7.06)	1.48(6.39)	1.38(5.97)	1.50(6.47)	1.61(6.89)	1.41(5.95)	1.37(5.98)
Delayed Gamma	1.48(5.61)	1.55(7.15)	1.46(5.72)	1.45(5.47)	1.68(7.90)	1.72(7.69)	1.52(5.71)	1.47(5.57)
House Gamma	1.39(4.73)	1.98(9.37)	1.67(16.92)	2.35(12.27)	4.00(21.60)	1.41(5.15)	1.40(4.79)	1.39(4.74)
Body Gamma	1.98(8.62)	2.18(9.41)	2.04(9.40)	14.33(35.62)	2.65(11.60)	2.04(9.40)	6.63(14.57)	2.19(10.80)

\*Neutron fluence multiplied by 50.0 to improve statistics on neutron components.

( ) values in parenthesis are results from the Dec 84 system marrow YCS leakage tape of  $\approx 30,000$  histories.

time was also investigated. Results showing the house exposure and FSD for a typical house leakage tape are presented in Table 15. The adjoint leakage data from these optimized houses were coupled with organ leakage tapes. Results are shown in Table 16. Perhaps the most important point to make from the results in Tables 15 and 16 is that the statistics of the house leakage data affect the statistics of the organ dose as it is derived from the in-house fluence. Furthermore, the most important house histories for the house exposure calculation may not be the most important histories for calculating the organ dose. It is probably not desirable to optimize the house tape for dose alone, since other parts of the house spectrum are responsible for body gamma-ray dose.

All of these biasing improvements require no modification in the dosimetry system. Thus, the changes made in the leakage tapes are transparent to the user of the system.