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NUMERICAL COMPARISON OF IMPROVED METHODS OF TESTING IN CONTINGENCY TABLES WITH SMALL FREQUENCIES

小度数の分割表に対する検定の改良に関する数値的比較

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小度数の分割表に対する検定の改良に関する数値的比較

Introduction and summary

The significance levels of various tests for a general $c\times k$ contingency table are usually given by large sample theory. But they are not accurate for the one having small frequencies. In this paper, a numerical evaluation was made to determine how good the approximation of significance level is for various improved tests that have been developed by Nass [1], Yoshimura [2], Gart [3] etc. for $c\times k$ contingency table with small frequencies in some of cells. For this purpose we compared the significance levels of the various approximate methods (i) with those of one-sided tail defined in terms of exact probabilities for given marginals in 2×2 table; (ii) with those of exact probabilities accumulated in the order of magnitude of χ^2 statistic or likelihood ratio (=LR) statistic in 2×3 table mentioned by Yates [4]. In 2×2 table it is well known that Yates' correction gives satisfactory result for small cell frequencies and the other methods that we have not referred here, can be considered if we devote our attention only to 2×2 or $2\times k$ table. But we are mainly interested in comparing the methods that are applicable to a general $c\times k$ table. It appears that such a comparison for the various improved methods in the same example has not been made explicitly, even though these tests are frequently used in biological and medical research.

Our numerical experience shows the following facts. The approximate significance levels due to Gart [3] and the modified Dandekar's method are somewhat better than the others for 2×2 tables, though they are somewhat too small for exact levels in the range one to five percent. The method of Gart [3] is conservative for 2×3 tables, but the others are not always conservative. The corrected LR statistic by Yoshimura [2] is always a little better than the LR statistic. However, the approximate significance level of the LR statistic was always smaller than the others except for tables having comparatively large marginals. The other methods that we investigated give almost the same results as the standard χ^2 test. The method of Nass [1] proved not to give as good an approximation as he conjectured.

The following notation of observed frequencies for a $c\times k$ contingency table is used throughout this paper.

緒言および要約

一般の $c\times k$ 分割表に対する種々の検定では，普通大標本論によって，有意水準が近似的に定められているが，小度数をもつ一般の分割表に対する有意水準では，よい近似を与えない．この論文ではあるCellのいくつかに小度数をもつ $c\times k$ 分割表に対して，Nass[1] 吉村[2]，Gart[3]，などによって改良された種々の検定法が，いかによい近似を与えるかを調べるために，数値的比較を行なった．棄却域の定め方として，2×2表の場合には，種々の近似法の有意水準とi)一定の周辺合計のもとでexact probalilitiesによって定義される有意水準(片側)と比較し，さらに，2×3表の場合には，Yates[4]によって示された分割表を使用して，種々の近似的有意水準と，ii) χ^2 統計量あるいは，尤度比(＝ＬＲ)統計量の大きさの順に累積したexact probabilitiesの有意水準とによって比較した．2×2表の場合には，Yatesの補正が小度数をもつ分割表に対してよい近似を与えることが，よく知られている．また2×2表あるいは2×k表においてのみ関心を向けるならば，この論文で引用しなかった他の近似法がいくつか考えられるが，一般の $c\times k$ 分割表に対して使用できる近似法におもな関心があるため，ここでは述べないことにする．このような一般の分割表に適用できる種々の近似法は，生物学および医学調査においてしばしば用いられているけれども，同じ例題に対して，改良された種々の近似法を適用したこのような数値的比較はまだ行なわれていないように思う．

数値的比較によって，2×2表の場合にはGart[3]と補正したDandekarの近似法によって得られた有意水準は，1％～5％の棄却域において，上述したexact probabilitiesの有意水準と比べて，あまりよい近似を与えていない．しかし，これらの近似法による有意水準は，他の近似法よりいくぶんよい近似を示している．Gart[3]の方法は，2×3表に対してすべてconservativeであるが，他の近似法は必ずしもconservativeでない．吉村[2]によって補正された尤度比統計量は，普通の尤度比統計量よりわずかであるが，常によい近似を与えている．しかしながら，尤度比統計量は他の近似法より，常に小さい有意水準を示しているが，比較的大きい周辺合計をもつ，2×2分割表に対しては，異なった傾向を示す。われわれが比較した他の近似法は普通のχ^2統計量とほとんど同じ結果を示した．Nass[1]の方法は，かれが推測したようには，よい近似を与えないことがわかった．

$c\times k$ 分割表に対する観測度数は，この論文を通じて次のような記号を用いる．

$$\begin{array}{c|c} X_{11}, X_{12}, \cdots, X_{1k} & X_{1\cdot} \\ X_{21}, X_{22}, \cdots, X_{2k} & X_{2\cdot} \\ \cdots\cdots\cdots\cdots\cdots & \cdots \\ X_{c1}, X_{c2}, \cdots, X_{ck} & X_{c\cdot} \\ \hline X_{\cdot 1}, X_{\cdot 2}, \cdots, X_{\cdot k} & N \end{array}$$

Description of methods with numerical comparisons in Rao's example

Rao ([5], p. 202) considered the following 2×2 table, testing whether the city soldiers are more sociable than village soldiers. In this example,

	Sociable	Nonsociable	Total
City soldiers	13	4	17
Village soldiers	6	14	20
Total	19	18	37

he compared the various improved techniques for small frequencies with the exact method, that is, the one sided tail probability ($X_{12}=0\sim 4$) for fixed marginals is given by equation (1) and the approximate significance levels of the other methods described by him are shown in (2)～(5).

種々の近似法とRaoの例題による数値的比較

Rao[5]は都会出身の軍人と町村出身の軍人について，社交性に差があるかどうかの検定を例にとり，次のような2×2表を考えた．

この例題において，Raoは小度数をもつ分割表に対して，改良した種々の近似法を精密検定と比較した．すなわち，一定の周辺合計のもとで，度数4のところを3,2,1,0,と動かしたときの片側検定($X_{12}=0\sim 4$)は，方程式(1)によって与えられる．さらに，Raoは他の方法(2)～(5)の近似的有意水準を示している．

(1) exact probability$=0.0059$
(2) χ^2 test, $P(\chi^2>7.9435)/2=0.0024$
(3) Yates' correction, $P(\chi^2>6.1922)/2=0.0064$
(4) LR test, $P(-2\log\lambda>8.2811)/2=0.0020$
(5) Dandekar's correction, $P(\chi^2>6.1086)/2=0.0068$.

(1) 精密検定$=0.0059$
(2) χ^2 検定, $P(\chi^2>7.9435)/2=0.0024$
(3) Yatesの補正, $P(\chi^2>6.1922)/2=0.0064$
(4) 尤度比検定, $P(-2\log\lambda>8.2811)/2=0.0020$
(5) Dandekarの補正, $P(\chi^2>6.1086)/2=0.0068$

These results show that χ^2 method (2) and LR method (4) do not give satisfactory approximation and that Yates' correction (3) and Dandekar's correction (5) are fairly good. But Yates' correction as well as Dandekar's cannot be extended to a general $c\times k$ table. We shall investigate some other techniques applicable to $c\times k$ tables and giving better approximations in this example.

これらの結果はχ^2検定(2)および，尤度比検定(4)も満足な近似を与えないことがわかる．一方，Yatesの補正(3)とDandekarの補正(5)は，かなりよい近似を与えている．しかしDandekerの補正と同様に，Yatesの補正も一般の $c\times k$ 分割表に適用することはできない．したがって，このRaoの例題に対して，よりよい近似を与え，しかも一般の$c\times k$ 分割表に対しても適用できる他のいくつかの方法を調べる．

(6) Corrected LR statistic. Yoshimura [2] calculated the correction factor K for the LR statistic, $-2\log\lambda$, such that the first and the second conditional moments for given marginals of the statistic, $-2K\log\lambda$, are equal to those of the $\chi^2_{(c-1)(k-1)}$ distribution up to the order of $1/N$, that is,

(6) 尤度比統計量の補正　吉村[2]は一定の周辺合計のもとで，統計量$-2K\log\lambda$の漸近的平均および分散が，$1/N$のorder まで極限分布$\chi^2_{(c-1)(k-1)}$ のそれらと一致するように，尤度比統計量 $-2\log\lambda$ に対する補正因子Kを計算した．

$$-2K\log\lambda=-2K\Big[\sum_{i=1}^{c}\sum_{j=1}^{k}X_{ij}\log X_{ij}-\sum_{i=1}^{c}X_{i.}\log X_{i.}-\sum_{j=1}^{k}X_{.j}\log X_{.j}+N\log N\Big]$$

where ただし

$$K=1-[6N(c-1)(k-1)]^{-1}\Big(N\sum_{i=1}^{c}X_{i.}^{-1}-1\Big)\Big(N\sum_{j=1}^{k}X_{.j}^{-1}-1\Big).$$

Then he proposed using the statistic $-2K\log\lambda$, which is approximately distributed according to the $\chi^2_{(c-1)(k-1)}$ distribution. In Rao's example, $K=0.95906$ and $P(-2K\log\lambda>7.9421)/2=0.0024$. The accuracy of this correction gives the same significance level as that of the χ^2 method (2), though it is a little better than the LR method (4).

この統計量，$-2K\log\lambda$ を使えば，　近似的に自由度 $(c-1)(k-1)$ のχ^2-分布に従うことを示した．Rao の例題について計算すれば，$K=0.95906$，$P(-2K\log\lambda>7.9421)/2=0.0024$となる．この補正法は尤度比検定(4)より，わずかによくなっているが，χ^2-検定(2)と同じ程度の近似である．

(7) Correction of χ^2 statistic. We shall determine the correction factors a and b such that the first and the second conditional moments of the statistic $a\chi^2+b$ are equal to those of $\chi^2_{(c-1)(k-1)}$ distribution. The conditional moments of the χ^2 statistic are given by Haldane [6].

(7) χ^2統計量に対する補正　統計量 $a\chi^2+b$ の漸近的平均および分散が，極限分布 $\chi^2_{(c-1)(k-1)}$ のそれらと一致するように，補正因子 a と b を定める．このχ^2統計量の漸近的積率は，Haldane[6]によって求められている．

$$E_1=E[\chi^2\,|\,X_{i.},X_{.j}]=N(N-1)^{-1}(c-1)(k-1),$$

$$E_2=E[\chi^4\,|\,X_{i.},X_{.j}]=N^3[(N-1)(N-2)(N-3)]^{-1}\cdot(A_0+A_1N^{-1}+A_2N^{-1}),$$

$$A_0=(c-1)^2(k-1)^2+2(c-1)(k-1),$$

$$A_1=-4(c-1)^2(k-1)^2+c^2k^2+2ck-2-(k^2+2k-2)N\sum_{i=1}^{c}X_{i.}^{-1}-(c^2+2c-2)N\sum_{j=1}^{k}X_{.j}^{-1}+N^2\Big(\sum_{i=1}^{c}X_{i.}^{-1}\Big)\Big(\sum_{j=1}^{k}X_{.j}^{-1}\Big),$$

$$A_2=ck(c-2)(k-2)+k(k-2)N\sum_{i=1}^{c}X_{i.}^{-1}+c(c-2)N\sum_{j=1}^{k}X_{.j}^{-1}+N^2\Big(\sum_{i=1}^{c}X_{i.}^{-1}\Big)\Big(\sum_{j=1}^{k}X_{.j}^{-1}\Big).$$

Then we get $a=[2(c-1)(k-1)/(E_2-E_1^2)]^{1/2}$ and $b=(c-1)(k-1)-aE_1$, regarding the statistic $a\chi^2+b$ as a $\chi^2_{(c-1)(k-1)}$ variate. In Rao's example, $a=0.9864$, $b=0.01382$ and $P(a\chi^2+b>7.8218)/2=0.0026$. This correction is slightly better than the χ^2 method (2), though it is not satisfactory to us.

とするとき，この統計量 $a\chi^2+b$ が $\chi^2_{(c-1)(k-1)}$ 分布に従うように，$a=[2(c-1)(k-1)/(E_2-E_1^2)]^{1/2}$ と $b=(c-1)(k-1)-aE_1$ を求めればよい．Raoの例題について計算すれば，$a=0.9864$，$b=0.01382$となり，$P(a\chi^2+b>7.8218)/2=0.0026$となる．したがってこの方法でも$\chi^2$検定がわずかによくなった程度でふじゅうぶんである．

(8) Method by Nass I. Nass [1] proposes to use the test statistic $a\chi^2$ as a χ^2_f variate, where the quantities a and f are determined such that the first and the second conditional moments of the statistic $a\chi^2$ are equal to those of the χ^2_f distribution, that is, $a=2E_1/(2E_2-E_1^2)$, $f=aE_1$ where E_1, E_2 are given by (7). This method differs from the others in that the test statistic has a χ^2 distribution with non-integer number of degrees of freedom. In order to get the tail probability for the χ^2_f distribution, we made use of the Chebyshev series expansions for gamma functions mentioned by Clenshaw [7] and the continued fraction expansion

(8) Nass の方法 I．Nass[1]は統計量$a\chi^2$の1次および2次の条件つき積率を，自由度fの χ^2 分布の条件つき積率と一致するように，定数aとfを定めて，Test統計量$a\chi^2$を，自由度fのχ^2統計量として用いることを提案している．すなわち，$a=2E_1/(2E_2-E_1^2)$，$f=aE_1$ で E_1 と E_2 は (7) に示されている．今までの方法は，すべて極限分布の自由度は固定して考えていたが，Nassの方法は，自由度も少し修正しようとするところが目新しい．自由度fの χ^2 分布の棄却域を計算するために，Clenshaw[7]によって述べられたガンマ関数Chebyshev級数展開および，GuptaとWaknis[8]に指適された連分数展開，

$$x^{-a}e^{x}\int_x^{\infty}e^{-x}x^{a-1}dx=\frac{1}{x+}\ \frac{1-a}{1+}\ \frac{1}{x+}\ \frac{2-a}{1+}\ \frac{2}{x+}\ \frac{3-a}{1+}\cdots\frac{n-1}{x+}\ \frac{n-a}{1+}\cdots$$

for $0<a<1$, which is discussed by Gupta and Waknis [8]. In Rao's example, $a=1.0000$, $f=1.0278$ and $P(a\chi_f^2>7.9439)/2=0.0025$. This result is almost the same as that for the χ^2 method (2) and method (7).

ただし，　$0<a<1$，を用いて計算した．Rao の例題に対して計算すれば，$a=1.0000$，$f=1.0278$，$P(a\chi_f^2>7.9439)/2=0.0025$，この結果は χ^2 検定(2)と，χ^2 近似法(7)に対する有意水準とほとんど同じ程度である．

(9) Method by Nass II. Nass [1] conjectured that his method (8) would be improved, if he could use the test statistic, $a\chi^2+b$, as if it were distributed according to the χ_f^2 distribution, where the parameters a, b and f are determined such that the first three conditional moments of $a\chi^2+b$ are equal to those of the χ_f^2 distribution. In a $2\times k$ table we can check this conjecture by using the third moment given by Haldane [9]. We shall write it after some rearrangement.

(9) Nassの方法Ⅱ．Nass[1]はtest統計量 $a\chi^2+b$ が自由度 f の χ^2 分布に従うとして，パラメーターを３つにして，その統計量の３次の条件つき積率まで一致させるように，a, b, f を定めれば，方法(8)よりよい近似を与えるであろうと予想した．$2\times k$ 分割表については，Haldane[9]より χ^2 の３次積率が求められているので，これを使ってNassの予想を調べてみることができる．
Haldane[9]の結果を書きかえると，

$$E_3=E[\chi^6\mid X_{i\cdot},X_{\cdot j}]=N^5[(N-1)(N-2)\cdots(N-5)]^{-1}\cdot\left\{(k^2-1)(k+3)+\sum_{n=1}^{5}c_nN^{-n}\right\},$$

$$c_1=2(30k^2+69k-43)-2(9k+47)N\sum_{j=1}^{k}X_{\cdot j}^{-1}+N^2(X_{1\cdot}X_{2\cdot})^{-1}\left\{-3k^3-21k^2-24k+26+(3k+19)N\sum_{j=1}^{k}X_{\cdot j}^{-1}\right\},$$

$$c_2=120(3k-1)-360N\sum_{j=1}^{k}X_{\cdot j}^{-1}+120N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}+N^2(X_{1\cdot}X_{2\cdot})^{-1}\left\{5k^3-57k^2-266k+120+3(9k+67)N\sum_{j=1}^{k}X_{\cdot j}^{-1}-30N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}\right\}$$
$$-N^4(X_{1\cdot}X_{2\cdot})^{-2}\left\{-2(k^3+9k^2+14k-12)+(3k+22)N\sum_{j=1}^{k}X_{\cdot j}^{-1}-N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}\right\},$$

$$c_3=N^2(X_{1\cdot}X_{2\cdot})^{-1}\left\{60k(k-2)-30(k-6)N\sum_{j=1}^{k}X_{\cdot j}^{-1}-90N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}\right\}-N^4(X_{1\cdot}X_{2\cdot})^{-2}\left\{6k(k-2)(k+5)+2(3k+23)N\sum_{j=1}^{k}X_{\cdot j}^{-1}-16N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}\right\},$$

$$c_4=-N^4(X_{1\cdot}X_{2\cdot})^{-2}\left\{4k(k^2-4)-(21k-20)N\sum_{j=1}^{k}X_{\cdot j}^{-1}-11N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}\right\},$$

$$c_5=-4N^4(X_{1\cdot}X_{2\cdot})^{-2}\left\{(3k-4)N\sum_{j=1}^{k}X_{\cdot j}^{-1}+N^2\sum_{j=1}^{k}X_{\cdot j}^{-2}\right\}.$$

By using the moments E_3 together with E_1, E_2 given by (7), we can determine the three parameters as $a=4m_2/m_3$, $b=4m_2(2m_2^2-m_1m_3)/m_3^2$, $f=8m_2^3/m_3^2$, where $m_1=E_1$, $m_2=E_2-E_1^2$ and $m_3=E_3-3E_2E_1+2E_1^3$. In Rao's example, $a=1.0287$, $b=0.03034$, $f=1.0877$ and $P(a\chi_f^2+b>8.2022)/2=0.0024$. The result of this correction is again the same as that of the χ^2 method (2) and methods (6)～(8). This approximation method conjectured by Nass does not seem to be satisfactory too.

(7)によって与えられた積率 E_1, E_2 とこの積率 E_3 を用いて，３つのパラメーター，$a=4m_2/m_3$,　$b=4m_2(2m_2^2-m_1m_3)/m_3^2$,　$f=8m_2^3/m_3^2$, ただし，$m_1=E_1$,　$m_2=E_2-E_1^2$,　$m_3=E_3-3E_2E_1+2E_1^3$．を決めることができる．Raoの例題に対して計算すれば，$a=1.0287$, $b=0.03034$, $f=1.0877$ で $P(a\chi_f^2+b>8.2022)/2=0.0024$ となり，χ^2 検定(2)と他の検定(6)～(8)の結果とほとんど同じである．Nassの予想した方法もそれほどよくないという結果になる．

(10) Method by Gart I. Gart [3] derived the modified LR statistic M/d, as a χ^2 variate with $(c-1)(k-1)$ degrees of freedom from an interesting viewpoint of regarding the data value X_{ij} as parameters and cell probabilities as random variables based on the equality connecting the multinomial distribution with the multivariate Beta distribution.

(10) Gartの方法Ⅰ．Gart[3]は多項分布を多変量ベータ分布に関連する等式をもとに，データ X_{ij} をパラメーターと考え，さらに，cellの確率を確率変数と考えることによって，興味ある尤度比統計量の補正 M/d が自由度 $(c-1)(k-1)$ の χ^2 統計量になるとした．

$$M=\sum_{i=1}^{c}\sum_{j=1}^{k}(2X_{ij}+1)\log(2X_{ij}+1)-\sum_{i=1}^{c}(2X_{i\cdot}+k)\log(2X_{i\cdot}+k)-\sum_{j=1}^{k}(2X_{\cdot j}+c)\log(2X_{\cdot j}+c)+(2N+ck)\log(2N+ck),$$

$$d=1+\frac{1}{3(c-1)(k-1)}\left\{\sum_{i=1}^{c}\sum_{j=1}^{k}\frac{1}{2X_{ij}+1}-\sum_{i=1}^{c}\frac{1}{2X_{i\cdot}+k}-\sum_{j=1}^{k}\frac{1}{2X_{\cdot j}+c}+\frac{1}{2N+ck}\right\}.$$

We shall remark that this correction factor d contains random variables X_{ij}, which distinguishes this method from others. In Rao's example, $M=7.8096$, $d=1.0565$ and $P(M/d>7.3920)/2=0.0033$. This result is somewhat better than the previous ones.

この方法は，補正因子 d の中に確率変数を含んでいる点で他の方法と異なっている．Raoの例題に対して計算すれば，$M=7.8096$, $d=1.0565$ となり，$P(M/d>7.3920)/2=0.0033$ となる．この結果は，上述した他の方法よりいくぶんよい結果を示している．

(11) Method by Gart II. Gart [3] also proposed to use the more accurate correction factor d' instead of d in the method (10).

⑾ Gartの方法Ⅱ．Gart[3]はさらに方法⑽で用いた補正因子 d の代わりに，より正確な補正因子 d' を用いることによって，結果がよくなると提案している

$$d'=\frac{1}{(c-1)(k-1)}\Big[\sum_{i=1}^{c}\sum_{j=1}^{k}\Big\{1-\frac{1}{3(2X_{ij}+1)}+\frac{1}{8(2X_{ij}+1)^2}\Big\}^{-1}$$
$$-\sum_{i=1}^{c}\Big\{1-\frac{1}{3(2X_{i\cdot}+k)}+\frac{1}{8(2X_{i\cdot}+k)^2}\Big\}^{-1}$$
$$-\sum_{j=1}^{k}\Big\{1-\frac{1}{3(2X_{\cdot j}+c)}+\frac{1}{8(2X_{\cdot j}+c)^2}\Big\}^{-1}$$
$$+\Big\{1-\frac{1}{3(2N+ck)}+\frac{1}{8(2N+ck)^2}\Big\}^{-1}\Big].$$

In Rao's example, $d'=1.0562$ and $P(M/d'>7.3944)/2=0.0033$. The result of this correction is the same as that of the method (10). The differences between methods (10) and (11) will be seen in another example later.

Raoの例題に対して計算すれば，$d'=1.0562$ で $P(M/d'>7.3944)/2=0.0033$ となる．この結果は方法⑽の結果と同じである．方法⑽と⑾の違いは，後に他の例題で調べる．

(12) Modified Dandekar's method. Dandekar's correction mentioned in Rao ([5], p. 203) cannot be extended to a general $c\times k$ table. This is because for given marginals we cannot uniquely increase or decrease the observed frequency by 1 according as the minimum observed frequency is decreased or increased by 1. We shall consider the following modification suggested by Mr. Ueda. Let the minimum number of all the cell frequencies be attained by the $(i_1,j_1),\cdots,(i_l,j_l)$ cells. Then we calculate the χ^2 statistic for the table having the modified frequency X'_{rs}, where $X'_{rs}=X_{rs}\pm l^{-1}$, $X'_{r\cdot}=X_{r\cdot}\pm$(number of $i_1,\cdots,i_l$ equal to $r)\times l^{-1}$, $X'_{\cdot s}=X_{\cdot s}\pm$(number of $j_1,\cdots,j_l$ equel to $s)\times l^{-1}$ for $(r,s)=(i_1,j_1),\cdots,(i_l,j_l)$, $X'_{rs}=X_{rs}$ for other cells and $N'=N\pm 1$. We shall define this value as $\chi^2_{\mp 1}$. The modified Dandekar's method is to use the statistic $\chi^2_c=\chi^2_0-|(\chi^2_0-\chi^2_{-1})(\chi^2_{+1}-\chi^2_0)/(\chi^2_{+1}-\chi^2_{-1})|$ as a χ^2 variate with $(c-1)(k-1)$ degrees of freedom, where χ^2_0 means the value of χ^2 statistic for the observed frequency X_{ij}. Here we must note that the inequalities $\chi^2_{+1}>\chi^2_0>\chi^2_{-1}$ are not necessarily hold. In fact, for $X_{12}=9\sim 12$ in figure 4 we have $\chi^2_{+1}<\chi^2_0<\chi^2_{-1}$. In Rao's example, $\chi^2_0=7.9435$, $\chi^2_{+1}=9.3678$, $\chi^2_{-1}=6.7556$ and $P(\chi^2_c>7.2958)/2=0.0035$. This result is slightly better than Gart's methods (10) and (11).

⑿ Dandekarの方法に対する補正．Rao[5] (p. 203) に述べてあるDandekarの方法は，一般の $c\times k$ 分割表に適用できない．これは周辺合計が一定のもとでは，一般の分割表の観測度数を１で加減することができないため，分割表の最小観測度数を１で加減する．われわれは，上田の示唆によって，次のような修正を考えた．まず，すべてのcell 度数の最小数を $(i_1,j_1),\cdots,(i_l,j_l)$ cellsとして与えられるとする．そこで加減した度数 X' の分割表に対して，χ^2 統計量を計算する．ただし $(r,s)=(i_1,j_1),\cdots,(i_l,j_l)$，において，$X'_{rs}=X_{rs}\pm l^{-1}$, $X'_{r\cdot}=X_{r\cdot}\pm$（$i_1,\cdots,i_l$ の数は r に等しい）$\times l^{-1}$，$X'_{\cdot s}=X_{\cdot s}\pm$（$j_1,\cdots,j_l$ の数は s に等しい）$\times l^{-1}$，他の cell に対して $X'_{rs}=X_{rs}$ および $N'=N\pm 1$ となる．この値は $\chi^2_{\mp 1}$ として定義される．補正したDandekarの方法は，自由度（$c-1$）（$k-1$）をもつ χ^2 統計量として，この統計量 $\chi^2_c=\chi^2_0-|(\chi^2_0-\chi^2_{-1})\ (\chi^2_{+1}-\chi^2_0)\ /\ (\chi^2_{+1}-\chi^2_{-1})|$ を用いる．ただし，χ^2_0 は観測度数 X_{ij} の χ^2 統計量を意味する．ここで注意せねばならないことは，不等式が必ずしも $\chi^2_{+1}>\chi^2_0>\chi^2_{-1}$ の順にならないことである．事実，図４の $X_{12}=9\sim 12$ において，不等式は $\chi^2_{+1}<\chi^2_0<\chi^2_{-1}$ のように逆になっている．Raoの例題を計算すれば，$\chi^2_0=7.9435$, $\chi^2_{+1}=9.3678$, $\chi^2_{-1}=6.7556$ となり，$P(\chi^2_c>7.2958)/2=0.0035$ となる．この結果はGartの方法⑽と⑾よりわずかよくよくなっている．

Comparison in some 2×2 and 2×3 tables

2×2および2×3表のいくつかの比較

In the previous section we have checked the accuracy for the various approximation methods in Rao's example only. We shall first investigate the effects of these corrections for $X_{12}=0\sim 7$ or $X_{11}=0\sim 8$ (accumulated in opposite direction) in the same example, whose results are shown in figures 1 and 2. Then we also examine in case of other tables as follows:

前の章ではRaoの例題だけを取り扱って種々の近似法の精度について調べた．ここでは，まず同じRaoの例題に対して周辺合計を一定として，$X_{12}=0\sim 7$ または $X_{11}=0\sim 8$（反対方向）へとcellの度数を増していった場合に，これらの近似法による結果を調べてみる．これらの結果は，図１と２（表１と２）に示している．さらに，次のような他の分割表が調べられる．

(i) Rao's example (Figures 1 and 2, Tables 1 and 2)

i）Raoの例題　図１と２（表１と２）

X_{11}	X_{12}	17
X_{21}	X_{22}	20
19	18	37

,

(ii) 2×2 symmetric case (Figure 3, Table 3)

ii）対称な２×２表　図３（表３）

X_{11}	X_{12}	20
X_{21}	X_{22}	20
20	20	40

,

(iii) 2×2 table with large marginals (Figure 4, Table 4)

iii）周辺合計が大きい２×２表　図４（表４）

X_{11}	X_{12}	18
X_{21}	X_{22}	555
133	440	573

,

(iv) 2×3 table by Yates ([4], p. 233) (Figures 5 and 6, Tables 5 and 6)

iv) Yates[4] (P.233) による2×3表（図5・6，表5・6）

X_{11}	X_{12}	X_{13}	17
X_{21}	X_{22}	X_{23}	13
13	11	6	30

Figures 1 and 2 show the following facts. Gart (10), (11) and modified Dandekar (12) give fairly better results and the LR method (4) always gives the smallest values. Yoshimura (6) is uniformly better than (4). The methods (7)∼(9) are almost the same as the χ^2 method (2). We must note in figure 3 that the value obtained by Gart (10) for $X_{12}=0$ is larger than for $X_{12}=1$. This fact seems to be a weak point of method (10). Gart (11) is preferable to Gart (10) in the presence of zero frequencies for some cells; otherwise they are nearly equal. Figure 4 shows that the LR method (4) and Yoshimura (6) are better and they are almost equal to Gart (10) and (11). In this case, the χ^2 method (2) as well as modified Dandekar's method (12) do not give good approximations. In figures 5 and 6, the methods (2) and (7)∼(9) give almost the same results as in the previous figures, but they are close to exact method (1). Gart (10) and (11) are conservative, but modified Dandekar (12) is not (See Tables 1 − 6).

図1と2(表1と2)は次のようなことを示している．Gartの方法(10)(11)と補正したDandekarの方法(12)は，いずれもかなりよい結果を示しており，尤度比(4)は，常に最も小さい統計量を与える．吉村の方法(6)は，一様に尤度比(4)よりよい結果である．他の方法(7)∼(9)は，χ^2の方法(2)とほとんど同じ程度である．図3(表3)で注意せねばならないことは，Gart(10)の方法では対称な分割表の度数 $X_{12}=1$の結果よりも，$X_{12}=0$ の値の方が大きくなる．これはGartの方法(10)の欠陥である．分割表のあるcellに0度数がある場合，Gart(11)はGart(10)よりよくなっているが，他の点ではほとんど等しい．図4(表4)においては，尤度法(4)と尤度法の補正(6)は，Gart(10)と(11)の結果と比べてほとんど同じ程度までよくなってくる．この場合，χ^2 の方法(2)と，補正した Dandekarの方法(12)は，よい近似を与えていない．図5と6(表5と6) において，方法(2)と(7)∼(9)は，前の図(表)による結果とほとんで同じであるが，これらの方法は精密検定(1)に近い．Gart(10)と(11)はすべてconservativeであるが，補正した Dandekarの方法(12)は，常に conservativeでない．

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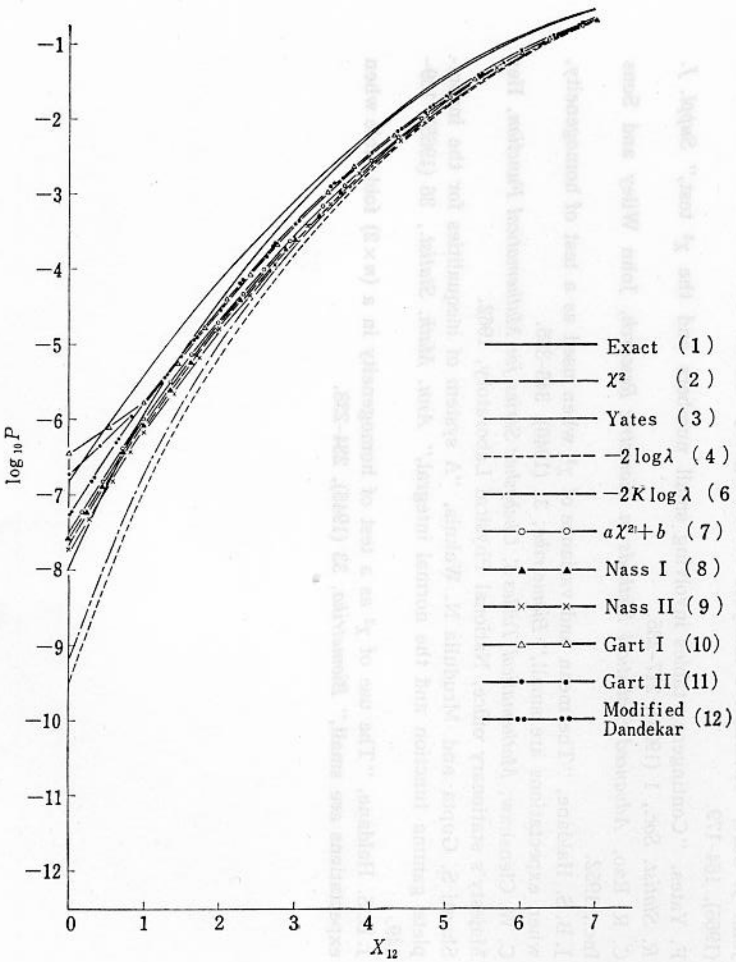

Fig. 1. One sided tail probabilities for $X_{12}=0\sim 7$ in Rao's example.

図1 Raoの例題の $X_{12}=0\sim 7$ としたときの片側確率

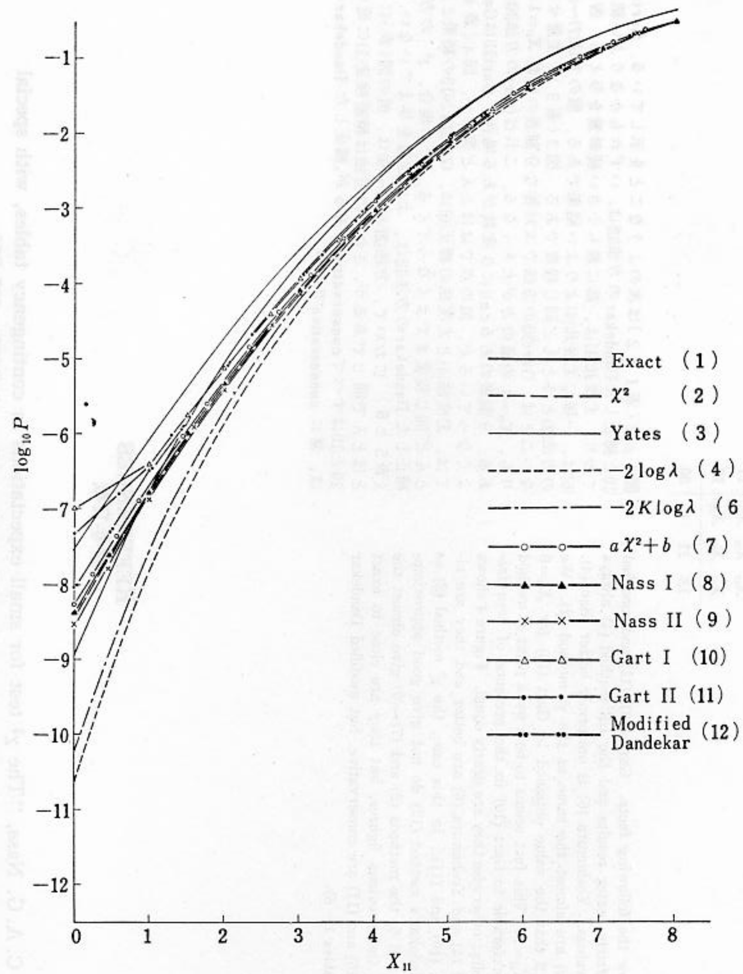

Fig. 2. One sided tail probabilities for $X_{11}=0\sim 8$ in Rao's example.

図2 Raoの例題の $X_{11}=0\sim 8$ としたときの片側確率

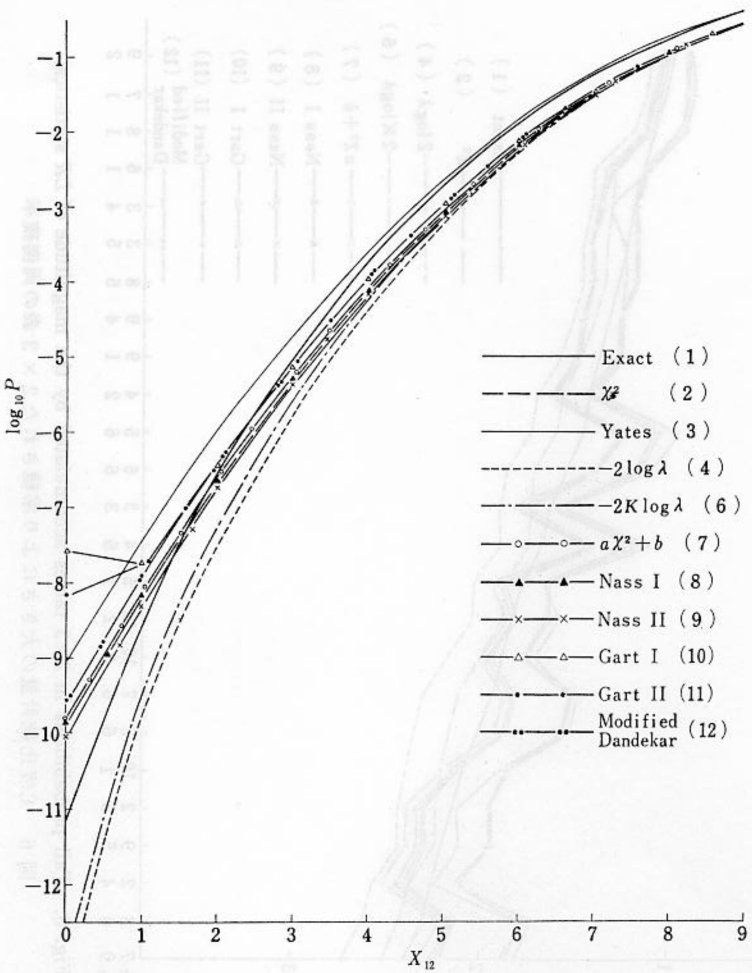

Fig. 3. One sided tail probabilities for $X_{12}=0\sim9$ in symmetric case.

図3 対称表の$X_{12}=0\sim9$としたときの片側確率

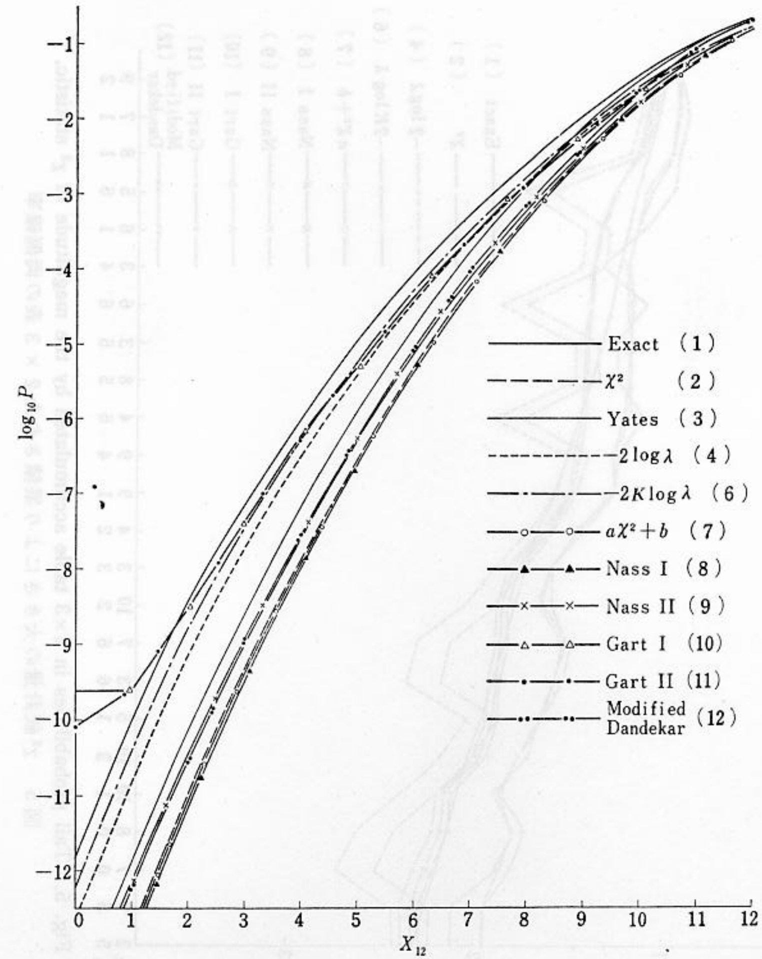

Fig. 4. One sided tail probabilities for $X_{12}=0\sim12$ with large marginals.

図4 大きい周辺合計表の$X_{12}=0\sim12$としたときの片側確率

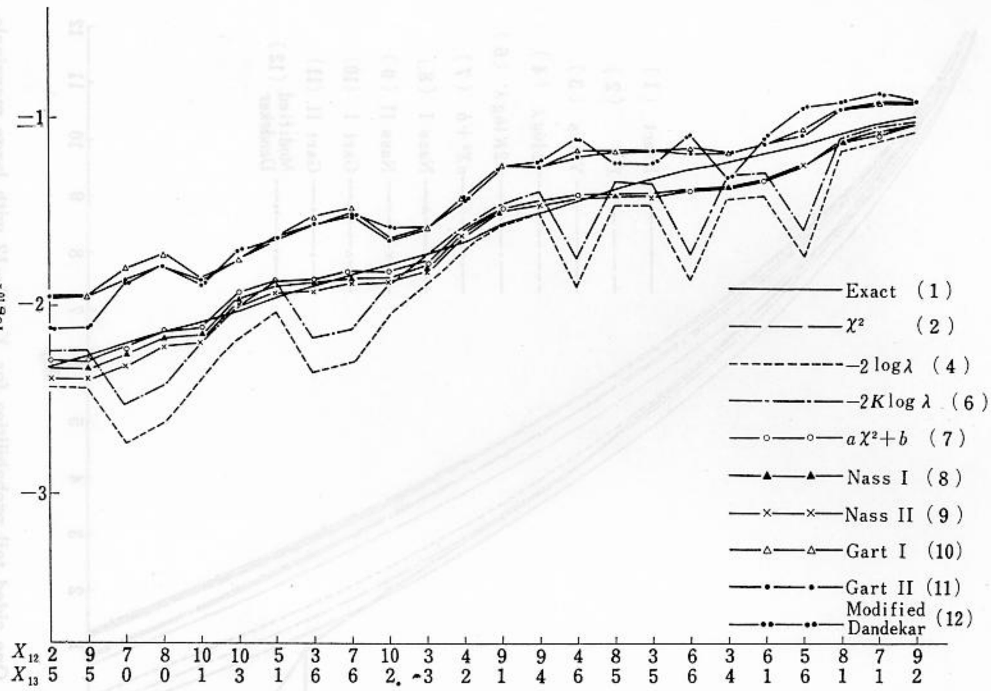

Fig. 5. Tail probabilities in 2×3 table accumulated by the magnitude of χ^2 statistic.
図５ χ^2統計量の大きさにより累積された２×３表の両側確率

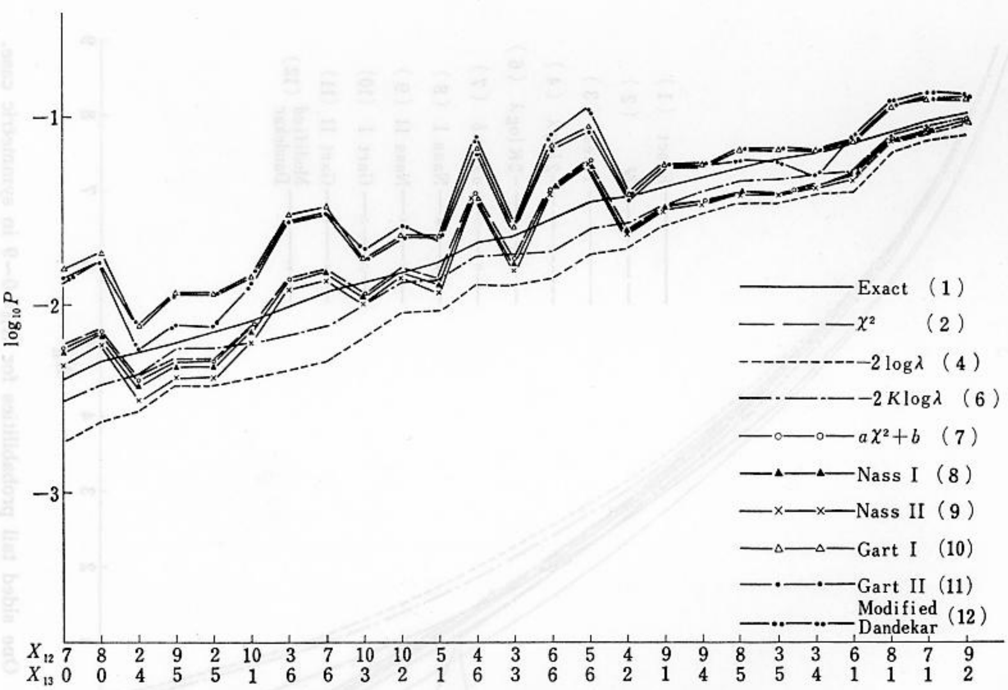

Fig. 6. Tail probabilities in 2×3 tables accumulated by the magnitude of *LR* statistics.
図６ 尤度比統計量の大きさにより累積された２×３表の両側確率

TABLE 1　ONE SIDED TAIL PROBABILITIES FOR $X_{12}=0\sim7$ IN RAO'S EXAMPLE

表1　Rao の例題の $X_{12}=0\sim7$ としたときの片側確率

X_{12}		Exact	Yates	χ^2	$-2\log\lambda$	$-2K\log\lambda$	Hald.	Nass I	Nass II	M/d	M/d'	χ_c^2
0	P	$.0^{7}1075$	$.0^{6}1461$	$.0^{7}2407$	$.0^{9}3104$	$.0^{9}6928$	$.0^{7}2987$	$.0^{7}2561$	$.0^{7}1864$	$.0^{6}3551$	$.0^{6}1785$	$.0^{7}5207$
	log	$\overline{8}.0314$	$\overline{7}.1647$	$\overline{8}.3815$	$\overline{10}.4919$	$\overline{10}.8406$	$\overline{8}.4752$	$\overline{8}.4084$	$\overline{8}.2704$	$\overline{7}.5504$	$\overline{7}.2516$	$\overline{8}.7166$
1	P	$.0^{5}1107$	$.0^{5}3940$	$.0^{6}8005$	$.0^{6}1157$	$.0^{6}2040$	$.0^{6}9487$	$.0^{6}8501$	$.0^{6}6768$	$.0^{5}1690$	$.0^{5}1627$	$.0^{5}1544$
	log	6.0441	6.5955	7.9034	$\overline{7}.0633$	$\overline{7}.3096$	$\overline{7}.9771$	$\overline{7}.9295$	$\overline{7}.8305$	6.2279	6.2114	6.1886
2	P	$.0^{4}3839$	$.0^{4}6993$	$.0^{4}1749$	$.0^{5}6764$	$.0^{4}1016$	$.0^{4}1992$	$.0^{4}1852$	$.0^{4}1592$	$.0^{4}2877$	$.0^{4}2851$	$.0^{4}3031$
	log	$\overline{5}.5842$	$\overline{5}.8447$	$\overline{5}.2428$	$\overline{6}.8302$	$\overline{5}.0069$	$\overline{5}.2993$	$\overline{5}.2676$	$\overline{5}.3019$	$\overline{5}.4589$	$\overline{5}.4550$	5.4816
3	P	$.0^{3}6349$	$.0^{3}8208$	$.0^{3}2522$	$.0^{3}1618$	$.0^{3}2148$	$.0^{3}2775$	$.0^{3}2660$	$.0^{3}2435$	$.0^{3}3757$	$.0^{3}3744$	$.0^{3}3957$
	log	$\overline{4}.8027$	$\overline{4}.9142$	$\overline{4}.4017$	$\overline{4}.2090$	$\overline{4}.3320$	$\overline{4}.4433$	$\overline{4}.4249$	$\overline{4}.3865$	$\overline{4}.5748$	$\overline{4}.5733$	$\overline{4}.5974$
4	P	$.0^{2}5855$	$.0^{2}6416$	$.0^{2}2413$	$.0^{2}2003$	$.0^{2}2415$	$.0^{2}2581$	$.0^{2}2533$	$.0^{2}2435$	$.0^{2}3276$	$.0^{2}3271$	$.0^{2}3456$
	log	$\overline{3}.7675$	$\overline{3}.8073$	$\overline{3}.3826$	$\overline{3}.3013$	$\overline{3}.3829$	$\overline{3}.4118$	$\overline{3}.4046$	$\overline{3}.3865$	$\overline{3}.5153$	$\overline{3}.5147$	$\overline{3}.5386$
5	P	$.0^{1}3300$	$.0^{1}3374$	$.0^{1}1545$	$.0^{1}1447$	$.0^{1}1622$	$.0^{1}1616$	$.0^{1}1612$	$.0^{1}1603$	$.0^{1}1905$	$.0^{1}1904$	$.0^{1}2037$
	log	$\overline{2}.5185$	$\overline{2}.5281$	$\overline{2}.1889$	$\overline{2}.1605$	$\overline{2}.2101$	$\overline{2}.2084$	$\overline{2}.2074$	$\overline{2}.2049$	$\overline{2}.2799$	$\overline{2}.2797$	$\overline{2}.3090$
6	P	.1212	.1213	$.0^{1}6701$	$.0^{1}6591$	$.0^{1}69999$	$.0^{1}6897$	$.0^{1}6941$	$.0^{1}7030$	$.0^{1}7563$	$.0^{1}7561$	$.0^{1}8212$
	log	$\overline{1}.0835$	$\overline{1}.0839$	$\overline{2}.8261$	$\overline{2}.8190$	$\overline{2}.8450$	$\overline{2}.8383$	$\overline{2}.8414$	$\overline{2}.8470$	$\overline{2}.8787$	$\overline{2}.8786$	$\overline{2}.9144$
7	P	.3060	.3056	.2009	.2005	.2054	.2049	.2060	.2084	.2115	.2115	.2313
	log	$\overline{1}.4857$	$\overline{1}.4852$	$\overline{1}.3030$	$\overline{1}.3021$	$\overline{1}.3126$	$\overline{1}.3115$	$\overline{1}.3139$	$\overline{1}.3189$	$\overline{1}.3253$	$\overline{1}.3253$	$\overline{1}.3642$

Note :　$.0^{7}1075 = .00000001075$
注　D F of Nass I = 1.0278
D F of Nass II = 1.0877
P = Probability　確率

TABLE 2　ONE SIDED TAIL PROBABILITIES FOR $X_{11}=0\sim8$ IN RAO'S EXAMPLE

表2　Rao の例題の $X_{11}=0\sim8$ としたときの片側確率

X_{11}		Exact	Yates	χ^2	$-2\log\lambda$	$-2K\log\lambda$	Hald.	Nass I	Nass II	M/d	M/d'	χ_c^2
0	P	$.0^{8}1132$	$.0^{7}2791$	$.0^{8}4180$	$.0^{10}2330$	$.0^{10}5767$	$.0^{8}5303$	$.0^{8}4447$	$.0^{8}3091$	$.0^{6}1023$	$.0^{7}4804$	$.0^{8}9050$
	log	$\overline{9}.0539$	$\overline{8}.4458$	$\overline{9}.6212$	$\overline{11}.3674$	$\overline{11}.7610$	$\overline{9}.7245$	$\overline{9}.6481$	$\overline{9}.4901$	$\overline{7}.0099$	$\overline{8}.6816$	$\overline{9}.9566$
1	P	$.0^{6}1839$	$.0^{6}9134$	$.0^{6}1686$	$.0^{7}1540$	$.0^{7}2942$	$.0^{6}2039$	$.0^{6}1792$	$.0^{6}1372$	$.0^{6}4071$	$.0^{6}3883$	$.0^{6}3273$
	log	$\overline{7}.2646$	$\overline{7}.9607$	$\overline{7}.2269$	$\overline{8}.1875$	$\overline{8}.4686$	$\overline{7}.3094$	$\overline{7}.2533$	$\overline{7}.1374$	$\overline{7}.6097$	$\overline{7}.5892$	$\overline{7}.5149$
2	P	$.0^{5}8957$	$.0^{4}1964$	$.0^{5}4467$	$.0^{5}1334$	$.0^{5}2135$	$.0^{5}5176$	$.0^{5}4736$	$.0^{5}3937$	$.0^{5}7812$	$.0^{5}7718$	$.0^{5}7823$
	log	$\overline{6}.9522$	$\overline{5}.2931$	$\overline{6}.6500$	$\overline{6}.1252$	$\overline{6}.3294$	$\overline{6}.7140$	$\overline{6}.6754$	$\overline{6}.5952$	6.8928	$\overline{6}.8875$	6.8934
3	P	$.0^{3}1954$	$.0^{3}2786$	$.0^{4}7791$	$.0^{4}4336$	$.0^{4}6056$	$.0^{4}8702$	$.0^{4}8233$	$.0^{4}7334$	$.0^{3}1220$	$.0^{3}1215$	$.0^{3}1240$
	log	$\overline{4}.2909$	$\overline{4}.4450$	$\overline{5}.8916$	$\overline{5}.6371$	$\overline{5}.7822$	$\overline{5}.9396$	$\overline{5}.9156$	$\overline{5}.8653$	$\overline{4}.0864$	$\overline{4}.0846$	$\overline{4}.09342$
4	P	$.0^{2}2283$	$.0^{2}2622$	$.0^{3}8992$	$.0^{3}6938$	$.0^{3}8708$	$.0^{3}9737$	$.0^{3}9461$	$.0^{3}8908$	$.0^{2}1282$	$.0^{2}1279$	$.0^{2}1309$
	log	$\overline{3}.3585$	$\overline{3}.4186$	$\overline{4}.9539$	$\overline{4}.8412$	$\overline{4}.9399$	$\overline{4}.9884$	$\overline{4}.9759$	$\overline{4}.9498$	3.1079	$\overline{3}.1069$	$\overline{3}.1169$
5	P	$.0^{1}1585$	$.0^{1}1652$	$.0^{2}6915$	$.0^{2}6265$	$.0^{2}7239$	$.0^{2}7302$	$.0^{2}7236$	$.0^{2}7098$	$.0^{2}8924$	$.0^{2}8917$	$.0^{2}9287$
	log	$\overline{2}.1999$	$\overline{2}.2180$	$\overline{3}.8398$	$\overline{3}.7969$	$\overline{3}.8597$	$\overline{3}.8634$	$\overline{3}.8595$	$\overline{3}.8511$	$\overline{3}.9506$	$\overline{3}.9502$	$\overline{3}.9679$
6	P	$.0^{1}7014$	$.0^{1}7056$	$.0^{1}3580$	$.0^{1}3478$	$.0^{1}3776$	$.0^{1}3709$	$.0^{1}3721$	$.0^{1}3746$	$.0^{1}4198$	$.0^{1}4196$	$.0^{1}4471$
	log	$\overline{2}.8460$	$\overline{2}.8486$	$\overline{2}.5539$	$\overline{2}.5413$	$\overline{2}.5770$	$\overline{2}.5693$	$\overline{2}.5707$	$\overline{2}.5736$	$\overline{2}.6230$	$\overline{2}.6228$	$\overline{2}.6504$
7	P	.2088	.2085	.1268	.1261	.1311	.1297	.1307	.1326	.1372	.1372	.1486
	log	$\overline{1}.3197$	$\overline{1}.3191$	$\overline{1}.1031$	$\overline{1}.1007$	$\overline{1}.1176$	$\overline{1}.1129$	$\overline{1}.1163$	$\overline{1}.1225$	$\overline{1}.1374$	$\overline{1}.1374$	$\overline{1}.1720$
8	P	.4398	.4397	.3150	.3150	.3185	.3214	.3207	.3196	.3225	.3224	.3503
	log	$\overline{1}.6433$	$\overline{1}.6432$	$\overline{1}.4983$	$\overline{1}.4983$	$\overline{1}.5031$	$\overline{1}.5070$	$\overline{1}.5061$	$\overline{1}.5046$	$\overline{1}.5085$	$\overline{1}.5084$	$\overline{1}.5444$

Note :　D F of Nass I = 1.0278
注　D F of Nass II = 1.0877

TABLE 3 ONE SIDED TAIL PROBABILITIES FOR $X_{12}=0\sim 9$ IN SYMMETRIC CASES

表3 対称表のX_{12}=0~9としたときの片側確率

X_{12}		Exact	Yates	χ^2	$-2\log\lambda$	$-2K\log\lambda$	Hald.	Nass I	Nass II	M/d	M/d'	χ_c^2
0	P	$.0^{11}7254$	$.0^{9}9372$	$.0^{9}1275$	$.0^{13}4789$	$.0^{12}1380$	$.0^{9}1672$	$.0^{9}1370$	$.0^{10}9129$	$.0^{7}2850$	$.0^{8}6983$	$.0^{9}2748$
	log	$\overline{12}.8606$	$\overline{10}.9718$	$\overline{10}.1055$	$\overline{14}.6802$	$\overline{13}.1399$	$\overline{10}.2232$	$\overline{10}.1367$	$\overline{11}.9604$	$\overline{8}.4548$	$\overline{9}.8440$	$\overline{10}.4390$
1	P	$.0^{8}2909$	$.0^{7}3811$	$.0^{8}6292$	$.0^{9}1582$	$.0^{9}3384$	$.0^{8}7854$	$.0^{8}6736$	$.0^{8}4918$	$.0^{7}1936$	$.0^{7}1772$	$.0^{7}1231$
	log	$\overline{9}.4637$	$\overline{8}.5810$	$\overline{9}.7988$	$\overline{10}.1992$	$\overline{10}.5294$	$\overline{9}.8951$	$\overline{9}.8284$	$\overline{9}.6918$	$\overline{8}.2869$	$\overline{8}.2485$	$\overline{8}.0903$
2	P	$.0^{6}2648$	$.0^{5}1051$	$.0^{6}2106$	$.0^{7}2876$	$.0^{7}5086$	$.0^{6}2513$	$.0^{6}2243$	$.0^{6}1777$	$.0^{6}3662$	$.0^{6}3584$	$.0^{6}3759$
	log	$\overline{7}.4229$	$\overline{6}.0216$	$\overline{7}.3235$	$\overline{8}.4588$	$\overline{8}.7064$	$\overline{7}.4002$	$\overline{7}.3508$	$\overline{7}.2497$	$\overline{7}.5637$	$\overline{7}.5544$	$\overline{7}.5751$
3	P	$.0^{5}9693$	$.0^{4}1970$	$.0^{5}4778$	$.0^{5}1649$	$.0^{5}2518$	$.0^{5}5490$	$.0^{5}5073$	$.0^{5}4309$	$.0^{5}7438$	$.0^{5}7382$	$.0^{5}7836$
	log	$\overline{6}.9865$	$\overline{5}.2945$	$\overline{6}.6792$	$\overline{6}.2172$	$\overline{6}.4011$	$\overline{6}.7396$	$\overline{6}.7053$	$\overline{6}.6344$	$\overline{6}.8715$	$\overline{6}.8682$	$\overline{6}.8941$
4	P	$.0^{3}1800$	$.0^{3}2521$	$.0^{4}7393$	$.0^{4}4305$	$.0^{4}5847$	$.0^{4}8214$	$.0^{4}7812$	$.0^{4}7039$	$.0^{3}1089$	$.0^{3}1086$	$.0^{3}1119$
	log	$\overline{4}.2553$	$\overline{4}.4016$	$\overline{5}.8688$	$\overline{5}.6340$	$\overline{5}.7669$	$\overline{5}.9146$	$\overline{5}.8928$	$\overline{5}.8475$	$\overline{4}.0370$	$\overline{4}.0358$	$\overline{4}.0488$
5	P	$.0^{2}1924$	$.0^{2}2213$	$.0^{3}7828$	$.0^{3}6083$	$.0^{3}7525$	$.0^{3}8434$	$.0^{3}8228$	$.0^{3}7776$	$.0^{2}1079$	$.0^{2}1078$	$.0^{2}1100$
	log	$\overline{3}.2842$	$\overline{3}.3450$	$\overline{4}.8937$	$\overline{4}.7841$	$\overline{4}.8765$	$\overline{4}.9260$	$\overline{4}.9153$	$\overline{4}.8908$	$\overline{3}.0330$	$\overline{3}.0326$	$\overline{3}.0414$
6	P	$.0^{1}1282$	$.0^{1}1343$	$.0^{2}5706$	$.0^{2}5149$	$.0^{2}5916$	$.0^{2}6020$	$.0^{2}5962$	$.0^{2}5842$	$.0^{2}7258$	$.0^{2}7253$	$.0^{2}7483$
	log	$\overline{2}.1079$	$\overline{2}.1281$	$\overline{3}.7563$	$\overline{3}.7117$	$\overline{3}.7720$	$\overline{3}.7796$	$\overline{3}.7754$	$\overline{3}.7666$	$\overline{3}.8608$	$\overline{3}.8605$	$\overline{3}.8741$
7	P	$.0^{1}5642$	$.0^{1}5692$	$.0^{1}2889$	$.0^{1}2793$	$.0^{1}3034$	$.0^{1}2993$	$.0^{1}2999$	$.0^{1}3009$	$.0^{1}3383$	$.0^{1}3382$	$.0^{1}3559$
	log	$\overline{2}.7514$	$\overline{2}.7553$	$\overline{2}.4607$	$\overline{2}.4461$	$\overline{2}.4820$	$\overline{2}.4761$	$\overline{2}.4770$	$\overline{2}.4784$	$\overline{2}.5293$	$\overline{2}.5292$	$\overline{2}.5513$
8	P	.1715	.1714	.1030	.1022	.1065	.1053	.1060	.1075	.1121	.1121	.1200
	log	$\overline{1}.2343$	$\overline{1}.2340$	$\overline{1}.0128$	$\overline{1}.0095$	$\overline{1}.0273$	$\overline{1}.0224$	$\overline{1}.0253$	$\overline{1}.0314$	$\overline{1}.0496$	$\overline{1}.0496$	$\overline{1}.0792$
9	P	.3762	.3759	.2635	.2634	.2673	.2681	.2687	.2678	.2720	.2720	.2930
	log	$\overline{1}.5754$	$\overline{1}.5751$	$\overline{1}.4208$	$\overline{1}.4206$	$\overline{1}.4270$	$\overline{1}.4283$	$\overline{1}.4293$	$\overline{1}.4310$	$\overline{1}.4346$	$\overline{1}.4346$	$\overline{1}.4669$

Note : D F of Nass I = 1.0249 D F of Nass II = 1.0780

TABLE 4 ONE SIDED TAIL PROBABILITIES FOR $X_{12}=0\sim 12$ WITH LARGE MARGINALS

表4 大きい周辺合計表のX_{12}=0~12としたときの片側確率

X_{12}		Exact	Yates	χ^2	$-2\log\lambda$	$-2K\log\lambda$	Hald.	Nass I	Nass II	M/d	M/d'	χ_c^2
0	P	$.0^{11}1502$	$.0^{13}2075$	$.0^{14}2236$	$.0^{13}7590$	$.0^{12}2486$	$.0^{14}1895$	$.0^{14}1567$	$.0^{13}1291$	$.0^{9}2517$	$.0^{10}8386$	$.0^{14}8523$
	log	$\overline{12}.1767$	$\overline{14}.3170$	$\overline{15}.3495$	$\overline{14}.8802$	$\overline{13}.3955$	$\overline{15}.2776$	$\overline{15}.1951$	$\overline{14}.1109$	$\overline{10}.4009$	$\overline{11}.9236$	$\overline{15}.9306$
1	P	$.0^{9}1040$	$.0^{11}1383$	$.0^{12}1749$	$.0^{10}1517$	$.0^{10}3979$	$.0^{12}1518$	$.0^{12}1293$	$.0^{12}7648$	$.0^{9}2475$	$.0^{9}2328$	$.0^{12}5737$
	log	$\overline{10}.01703$	$\overline{12}.1408$	$\overline{13}.2428$	$\overline{11}.1810$	$\overline{11}.5998$	$\overline{13}.1813$	$\overline{13}.1116$	$\overline{13}.8835$	$\overline{10}.3936$	$\overline{10}.3670$	$\overline{13}.7587$
2	P	$.0^{8}3374$	$.0^{10}6724$	$.0^{11}9976$	$.0^{9}6984$	$.0^{8}1561$	$.0^{11}8842$	$.0^{11}7745$	$.0^{10}3376$	$.0^{8}3075$	$.0^{8}3028$	$.0^{10}2840$
	log	$\overline{9}.5281$	$\overline{11}.8276$	$\overline{12}.9990$	$\overline{10}.8441$	$\overline{9}.1934$	$\overline{12}.9466$	$\overline{12}.8890$	$\overline{11}.5284$	$\overline{9}.4878$	$\overline{9}.4812$	$\overline{11}.4533$
3	P	$.0^{7}6812$	$.0^{8}2386$	$.0^{9}4150$	$.0^{7}1772$	$.0^{7}3462$	$.0^{9}3751$	$.0^{9}3369$	$.0^{8}1112$	$.0^{7}4260$	$.0^{7}4234$	$.0^{8}1034$
	log	$\overline{8}.8333$	$\overline{9}.3777$	$\overline{10}.6180$	$\overline{8}.2475$	$\overline{8}.5393$	$\overline{10}.5741$	$\overline{10}.5275$	$\overline{9}.04610$	$\overline{8}.6294$	$\overline{8}.6268$	$\overline{9}.0145$
4	P	$.0^{6}9597$	$.0^{7}6190$	$.0^{7}1261$	$.0^{6}2974$	$.0^{6}5170$	$.0^{7}1160$	$.0^{7}1061$	$.0^{7}2736$	$.0^{6}5050$	$.0^{6}5036$	$.0^{7}2776$
	log	$\overline{7}.9821$	$\overline{8}.7917$	$\overline{8}.1007$	$\overline{7}.4733$	$\overline{7}.7135$	$\overline{8}.06446$	$\overline{8}.02572$	$\overline{8}.4371$	$\overline{7}.7033$	$\overline{7}.7021$	$\overline{8}.4434$
5	P	$.0^{4}1003$	$.0^{5}1175$	$.0^{6}2800$	$.0^{5}3593$	$.0^{5}5638$	$.0^{6}2619$	$.0^{6}2456$	$.0^{6}5036$	$.0^{5}4883$	$.0^{5}4876$	$.0^{6}5495$
	log	$\overline{5}.0013$	$\overline{6}.07004$	$\overline{7}.4472$	$\overline{6}.5555$	$\overline{6}.7510$	$\overline{7}.4181$	$\overline{7}.3902$	$\overline{7}.7021$	$\overline{6}.6887$	$\overline{6}.6881$	$\overline{7}.7400$
6	P	$.0^{4}8068$	$.0^{4}1637$	$.0^{5}4562$	$.0^{4}3279$	$.0^{4}4700$	$.0^{5}4325$	$.0^{5}4127$	$.0^{5}6949$	$.0^{4}3829$	$.0^{4}3826$	$.0^{5}8041$
	log	$\overline{5}.9068$	$\overline{5}.2140$	$\overline{6}.6592$	$\overline{5}.5157$	$\overline{5}.6721$	$\overline{6}.6360$	$\overline{6}.6156$	$\overline{6}.8419$	$\overline{5}.5831$	$\overline{5}.5827$	$\overline{6}.9053$
7	P	$.0^{3}5115$	$.0^{3}1677$	$.0^{4}5442$	$.0^{3}2328$	$.0^{3}3083$	$.0^{4}5233$	$.0^{4}5070$	$.0^{4}7207$	$.0^{3}2445$	$.0^{3}2443$	$.0^{4}8717$
	log	$\overline{4}.7088$	$\overline{4}.2245$	$\overline{5}.7358$	$\overline{4}.3670$	$\overline{4}.4890$	$\overline{5}.7188$	$\overline{5}.7050$	$\overline{5}.8578$	$\overline{4}.3883$	$\overline{4}.3879$	$\overline{5}.9404$
8	P	$.0^{2}2597$	$.0^{2}1268$	$.0^{3}4834$	$.0^{2}1312$	$.0^{2}1622$	$.0^{3}4656$	$.0^{3}4568$	$.0^{3}5638$	$.0^{2}1278$	$.0^{2}1277$	$.0^{3}7022$
	log	$\overline{3}.4145$	$\overline{3}.1031$	$\overline{4}.6843$	$\overline{3}.1179$	$\overline{3}.2101$	$\overline{4}.6680$	$\overline{4}.6597$	$\overline{4}.7511$	$\overline{3}.1065$	$\overline{3}.1062$	$\overline{4}.8465$
9	P	$.0^{1}1067$	$.0^{2}7108$	$.0^{2}3115$	$.0^{2}5944$	$.0^{2}6925$	$.0^{2}3058$	$.0^{2}3031$	$.0^{2}3343$	$.0^{2}5497$	$.0^{2}5496$	$.0^{2}5250$
	log	$\overline{2}.02816$	$\overline{3}.8517$	$\overline{3}.4935$	$\overline{3}.7742$	$\overline{3}.8404$	$\overline{3}.4854$	$\overline{3}.4816$	$\overline{3}.5241$	$\overline{3}.7401$	$\overline{3}.7400$	$\overline{3}.7202$
10	P	$.0^{1}3572$	$.0^{1}2975$	$.0^{1}1507$	$.0^{1}2184$	$.0^{1}2421$	$.0^{1}1491$	$.0^{1}1489$	$.0^{1}1513$	$.0^{1}1953$	$.0^{1}1953$	$.0^{1}2376$
	log	$\overline{2}.5529$	$\overline{2}.4735$	$\overline{2}.1781$	$\overline{2}.3393$	$\overline{2}.3840$	$\overline{2}.1735$	$\overline{2}.1729$	$\overline{2}.1798$	$\overline{2}.2907$	$\overline{2}.2907$	$\overline{2}.3758$
11	P	$.0^{1}9790$	$.0^{1}9388$	$.0^{1}5470$	$.0^{1}6544$	$.0^{1}6971$	$.0^{1}5552$	$.0^{1}5463$	$.0^{1}5279$	$.0^{1}5746$	$.0^{1}5745$	$.0^{1}8025$
	log	$\overline{2}.9908$	$\overline{2}.9726$	$\overline{2}.7380$	$\overline{2}.8158$	$\overline{2}.8433$	$\overline{2}.7444$	$\overline{2}.7374$	$\overline{2}.7226$	$\overline{2}.7594$	$\overline{2}.7593$	$\overline{2}.9044$
12	P	.2204	.2266	.1507	.1606	.1659	.1531	.1514	.1448	.1403	.1403	.2052
	log	$\overline{1}.3432$	$\overline{1}.3553$	$\overline{1}.1781$	$\overline{1}.2057$	$\overline{1}.2198$	$\overline{1}.1850$	$\overline{1}.1801$	$\overline{1}.1608$	$\overline{1}.1471$	$\overline{1}.1471$	$\overline{1}.3122$

Note : D F of Nass I = 1.0298 D F of Nass II = .8606

TABLE 5 TAIL PROBABILITIES IN 2 × 3 TABLE ACCUMULATED BY THE MAGNITUDE OF χ^2 STATISTIC

表5 χ^2 統計量の大きさにより累積された2×3表の両側確率

x_{12}	x_{13}		Exact	χ^2	$-2\log\lambda$	$-2K\log\lambda$	Hald.	Nass I	Nass II	M/d	M/d'	χ^2_c
2	5	S		10.5446	11.1708	10.3094	10.5743	11.1638	12.5735	8.9570	8.9766	9.7528
		P	$.0^{2}4560$	$.0^{2}5132$	$.0^{2}3752$	$.0^{2}5772$	$.0^{2}5056$	$.0^{2}4722$	$.0^{2}4041$	$.0^{1}1135$	$.0^{1}1124$	$.0^{2}7624$
		log	$\bar{3}.6590$	$\bar{3}.7103$	$\bar{3}.5743$	$\bar{3}.7613$	$\bar{3}.7038$	$\bar{3}.6741$	$\bar{3}.6065$	$\bar{2}.05077$	$\bar{2}.05077$	$\bar{3}.8822$
9	5	S		10.5446	11.1708	10.3094	10.5743	11.1638	12.5735	8.9570	8.9766	9.7528
		P	$.0^{2}5347$	$.0^{2}5132$	$.0^{2}3752$	$.0^{2}5772$	$.0^{2}5056$	$.0^{2}4722$	$.0^{2}4041$	$.0^{1}1135$	$.0^{1}1124$	$.0^{2}7624$
		log	$\bar{3}.7281$	$\bar{3}.7103$	$\bar{3}.5743$	$\bar{3}.7613$	$\bar{3}.7038$	$\bar{3}.6741$	$\bar{3}.6065$	$\bar{2}.05077$	$\bar{2}.05077$	$\bar{3}.8822$
7	0	S		10.2361	12.5880	11.6174	10.2622	10.8372	12.2129	8.3059	8.5569	8.6297
		P	$.0^{2}6135$	$.0^{2}5988$	$.0^{2}1847$	$.0^{2}3001$	$.0^{2}5910$	$.0^{2}5546$	$.0^{2}4799$	$.0^{1}1572$	$.0^{1}1386$	$.0^{1}1337$
		log	$\bar{3}.7878$	$\bar{3}.7773$	$\bar{3}.2665$	$\bar{3}.4773$	$\bar{3}.7716$	$\bar{3}.7440$	$\bar{3}.6812$	$\bar{2}.1965$	$\bar{2}.1418$	$\bar{2}.1261$
8	0	S		9.8374	12.1147	11.1805	9.8589	10.4150	11.7470	7.9393	8.1793	8.2294
		P	$.0^{2}7120$	$.0^{2}7309$	$.0^{2}2341$	$.0^{2}3734$	$.0^{2}7231$	$.0^{2}6828$	$.0^{2}5990$	$.0^{1}1888$	$.0^{1}1675$	$.0^{1}1633$
		log	$\bar{3}.8525$	$\bar{3}.8639$	$\bar{3}.3694$	$\bar{3}.5722$	$\bar{3}.8592$	$\bar{3}.8343$	$\bar{3}.7774$	$\bar{2}.2760$	$\bar{2}.2240$	$\bar{2}.2130$
10	1	S		9.7472	11.0003	10.1521	9.7676	10.3196	11.6416	8.5363	8.5631	8.7135
		P	$.0^{2}8066$	$.0^{2}7646$	$.0^{2}4086$	$.0^{2}6245$	$.0^{2}7568$	$.0^{2}7156$	$.0^{2}6298$	$.0^{1}1401$	$.0^{1}1382$	$.0^{1}1282$
		log	$\bar{3}.9067$	$\bar{3}.8834$	$\bar{3}.6113$	$\bar{3}.7955$	$\bar{3}.8790$	$\bar{3}.8547$	$\bar{3}.7992$	$\bar{2}.1464$	$\bar{2}.1405$	$\bar{2}.1079$
10	3	S		8.9118	9.9859	9.2159	8.9226	9.4351	10.6654	8.0776	8.0938	7.8410
		P	$.0^{2}9380$	$.0^{1}1161$	$.0^{2}6786$	$.0^{2}9972$	$.0^{1}1155$	$.0^{1}1106$	$.0^{1}1001$	$.0^{1}1762$	$.0^{1}1748$	$.0^{1}1983$
		log	$\bar{3}.9722$	$\bar{2}.0648$	$\bar{3}.8316$	$\bar{3}.9988$	$\bar{2}.0626$	$\bar{2}.04376$	$\bar{2}.0004$	$\bar{2}.2460$	$\bar{2}.2425$	$\bar{2}.2973$
5	1	S		8.6080	9.3266	8.6074	8.6152	9.1135	10.3104	7.4971	7.5129	7.6196
		P	$.0^{1}1119$	$.0^{1}1351$	$.0^{2}9435$	$.0^{1}1352$	$.0^{1}1347$	$.0^{1}1295$	$.0^{1}1183$	$.0^{1}2355$	$.0^{1}2337$	$.0^{1}2215$
		log	$\bar{2}.0488$	$\bar{2}.1307$	$\bar{3}.9747$	$\bar{2}.1310$	$\bar{2}.1294$	$\bar{2}.1123$	$\bar{2}.07298$	$\bar{2}.3720$	$\bar{2}.3687$	$\bar{2}.3454$
3	6	S		8.5843	10.8397	10.0039	8.5912	9.0884	10.2827	6.9776	7.1887	7.1764
		P	$.0^{1}1296$	$.0^{1}1368$	$.0^{2}4428$	$.0^{2}6725$	$.0^{1}1363$	$.0^{1}1311$	$.0^{1}1199$	$.0^{1}3054$	$.0^{1}2748$	$.0^{1}2765$
		log	$\bar{2}.1126$	$\bar{2}.1361$	$\bar{3}.6462$	$\bar{3}.8277$	$\bar{2}.1345$	$\bar{2}.1176$	$\bar{2}.0788$	$\bar{2}.4849$	$\bar{2}.4390$	$\bar{2}.4417$
7	6	S		8.3565	10.5850	9.7688	8.3608	8.8472	10.0165	6.8108	7.0168	6.9497
		P	$.0^{1}1493$	$.0^{1}1533$	$.0^{2}5029$	$.0^{2}7564$	$.0^{1}1529$	$.0^{1}1476$	$.0^{1}1360$	$.0^{1}3319$	$.0^{1}2994$	$.0^{1}3097$
		log	$\bar{2}.1741$	$\bar{2}.1855$	$\bar{3}.7015$	$\bar{3}.8788$	$\bar{2}.1844$	$\bar{2}.1691$	$\bar{2}.1335$	$\bar{2}.5210$	$\bar{2}.4763$	$\bar{2}.4909$
10	2	S		8.3375	9.3905	8.6664	8.3416	8.8271	9.9943	7.5235	7.5395	7.2696
		P	$.0^{1}1670$	$.0^{1}1547$	$.0^{2}9139$	$.0^{1}1313$	$.0^{1}1544$	$.0^{1}1490$	$.0^{1}1374$	$.0^{1}2324$	$.0^{1}2306$	$.0^{1}2639$
		log	$\bar{2}.2227$	$\bar{2}.1895$	$\bar{3}.9609$	$\bar{2}.1183$	$\bar{2}.1886$	$\bar{2}.1732$	$\bar{2}.1380$	$\bar{2}.3662$	$\bar{2}.3629$	$\bar{2}.4214$
3	2	S		8.1144	8.6828	8.0133	8.1159	8.5909	9.7336	7.2820	7.2892	7.2712
		P	$.0^{1}1885$	$.0^{1}1730$	$.0^{1}1302$	$.0^{1}1819$	$.0^{1}1728$	$.0^{1}1674$	$.0^{1}1554$	$.0^{1}2623$	$.0^{1}2613$	$.0^{1}2637$
		log	$\bar{2}.2753$	$\bar{2}.2380$	$\bar{2}.1146$	$\bar{2}.2598$	$\bar{2}.2375$	$\bar{2}.2238$	$\bar{2}.1915$	$\bar{2}.4188$	$\bar{2}.4171$	$\bar{2}.4211$
4	2	S		7.3123	7.8327	7.2288	7.3044	7.7417	8.7962	6.5362	6.5431	6.6468
		P	$.0^{1}2207$	$.0^{1}2583$	$.0^{1}1991$	$.0^{1}2693$	$.0^{1}2593$	$.0^{1}2538$	$.0^{1}2413$	$.0^{1}3808$	$.0^{1}3795$	$.0^{1}3603$
		log	$\bar{2}.3438$	$\bar{2}.4121$	$\bar{2}.2991$	$\bar{2}.4302$	$\bar{2}.4138$	$\bar{2}.4045$	$\bar{2}.3826$	$\bar{2}.5807$	$\bar{2}.5792$	$\bar{2}.5567$
9	1	S		6.7854	7.2713	6.7106	6.7714	7.1839	8.1805	5.7680	5.7800	5.7893
		P	$.0^{1}2680$	$.0^{1}3362$	$.0^{1}2637$	$.0^{1}3490$	$.0^{1}3385$	$.0^{1}3335$	$.0^{1}3218$	$.0^{1}5591$	$.0^{1}5558$	$.0^{1}5532$
		log	$\bar{2}.4281$	$\bar{2}.5266$	$\bar{2}.4211$	$\bar{2}.5428$	$\bar{2}.5296$	$\bar{2}.5231$	$\bar{2}.5076$	$\bar{2}.7475$	$\bar{2}.7449$	$\bar{2}.7429$
9	4	S		6.6288	6.9364	6.4015	6.6130	7.0181	7.9975	5.7618	5.7679	6.1155
		P	$.0^{1}3173$	$.0^{1}3636$	$.0^{1}3117$	$.0^{1}4073$	$.0^{1}3665$	$.0^{1}3617$	$.0^{1}3504$	$.0^{1}5609$	$.0^{1}5591$	$.0^{1}4699$
		log	$\bar{2}.5015$	$\bar{2}.5606$	$\bar{2}.4937$	$\bar{2}.6099$	$\bar{2}.5641$	$\bar{2}.5583$	$\bar{2}.5446$	$\bar{2}.7489$	$\bar{2}.7475$	$\bar{2}.6720$
4	6	S		6.4769	8.6885	8.0185	6.4593	6.8573	7.8200	5.3686	5.5312	5.0796
		P	$.0^{1}3646$	$.0^{1}3922$	$.0^{1}1298$	$.0^{1}1815$	$.0^{1}3957$	$.0^{1}3913$	$.0^{1}3806$	$.0^{1}6827$	$.0^{1}6294$	$.0^{1}7888$
		log	$\bar{2}.5618$	$\bar{2}.5935$	$\bar{2}.1133$	$\bar{2}.2589$	$\bar{2}.5974$	$\bar{2}.5925$	$\bar{2}.5805$	$\bar{2}.8342$	$\bar{2}.7989$	$\bar{2}.8970$
8	5	S		6.4437	6.7079	6.1907	6.4257	6.8221	7.7811	5.3844	5.3946	5.6727
		P	$.0^{1}4237$	$.0^{1}3988$	$.0^{1}3495$	$.0^{1}4526$	$.0^{1}4024$	$.0^{1}3981$	$.0^{1}3876$	$.0^{1}6773$	$.0^{1}6739$	$.0^{1}5864$
		log	$\bar{2}.6271$	$\bar{2}.6008$	$\bar{2}.5434$	$\bar{2}.6557$	$\bar{2}.6047$	$\bar{2}.6000$	$\bar{2}.5884$	$\bar{2}.8308$	$\bar{2}.8286$	$\bar{2}.7682$
3	5	S		6.4437	6.7079	6.1907	6.4257	6.8221	7.7811	5.3844	5.3946	5.6727
		P	$.0^{1}4828$	$.0^{1}3988$	$.0^{1}3495$	$.0^{1}4526$	$.0^{1}4024$	$.0^{1}3981$	$.0^{1}3876$	$.0^{1}6773$	$.0^{1}6739$	$.0^{1}5864$
		log	$\bar{2}.6838$	$\bar{2}.6008$	$\bar{2}.5434$	$\bar{2}.6557$	$\bar{2}.6047$	$\bar{2}.6000$	$\bar{2}.5884$	$\bar{2}.8308$	$\bar{2}.8286$	$\bar{2}.7682$
6	6	S		6.3630	8.5725	7.9115	6.3441	6.7367	7.6868	5.2896	5.4498	4.9663
		P	$.0^{1}5325$	$.0^{1}4152$	$.0^{1}1376$	$.0^{1}1914$	$.0^{1}4192$	$.0^{1}4150$	$.0^{1}4049$	$.0^{1}7102$	$.0^{1}6555$	$.0^{1}8348$
		log	$\bar{2}.7263$	$\bar{2}.6183$	$\bar{2}.1386$	$\bar{2}.2819$	$\bar{2}.6224$	$\bar{2}.6180$	$\bar{2}.6073$	$\bar{2}.8514$	$\bar{2}.8166$	$\bar{2}.9216$

TABLE 5 (Continued) 続き

X_{12}	X_{13}		Exact	χ^2	$-2\log\lambda$	$-2K\log\lambda$	Hald.	Nass I	Nass II	M/d	M/d'	χ^2_c
3	4	S		6.2871	6.4795	5.9799	6.2672	6.6563	7.5981	5.4434	5.4484	6.0415
		P	$.0^{1}5916$	$.0^{1}4313$	$.0^{1}3917$	$.0^{1}5029$	$.0^{1}4356$	$.0^{1}4317$	$.0^{1}4220$	$.0^{1}6576$	$.0^{1}6560$	$.0^{1}4877$
		log	$\bar{2}.7720$	$\bar{2}.6348$	$\bar{2}.5930$	$\bar{2}.7015$	$\bar{2}.6391$	$\bar{2}.6352$	$\bar{2}.6253$	$\bar{2}.8180$	$\bar{2}.8169$	$\bar{2}.6882$
6	1	S		6.1020	6.4437	5.9468	6.0800	6.4603	7.3818	5.1827	5.1924	5.1029
		P	$.0^{1}6578$	$.0^{1}4731$	$.0^{1}3988$	$.0^{1}5113$	$.0^{1}4784$	$.0^{1}4750$	$.0^{1}4665$	$.0^{1}7492$	$.0^{1}7455$	$.0^{1}7797$
		log	$\bar{2}.8181$	$\bar{2}.6750$	$\bar{2}.6008$	$\bar{2}.7087$	$\bar{2}.6798$	$\bar{2}.6767$	$\bar{2}.6689$	$\bar{2}.8746$	$\bar{2}.8724$	$\bar{2}.8919$
5	6	S		5.7365	7.9509	7.3378	5.7102	6.0734	6.9547	4.8133	4.9592	4.3429
		P	$.0^{1}7240$	$.0^{1}5680$	$.0^{1}1877$	$.0^{1}2550$	$.0^{1}5755$	$.0^{1}5738$	$.0^{1}5684$	$.0^{1}9012$	$.0^{1}8378$	.1140
		log	$\bar{2}.8597$	$\bar{2}.7543$	$\bar{2}.2735$	$\bar{2}.4065$	$\bar{2}.7600$	$\bar{2}.7588$	$\bar{2}.7547$	$\bar{2}.9548$	$\bar{2}.9231$	$\bar{1}.05690$
8	1	S		5.1907	5.4330	5.0141	5.1580	5.4955	6.3168	4.3219	4.3300	4.1878
		P	$.0^{1}8304$	$.0^{1}7462$	$.0^{1}6611$	$.0^{1}8151$	$.0^{1}7585$	$.0^{1}7603$	$.0^{1}7623$	.1152	.1148	.1232
		log	$\bar{2}.9193$	$\bar{2}.8729$	$\bar{2}.8203$	$\bar{2}.9112$	$\bar{2}.8800$	$\bar{2}.8810$	$\bar{2}.8821$	$\bar{1}.0645$	$\bar{1}.05994$	$\bar{1}.0906$
7	1	S		4.9628	5.1783	4.7790	4.9276	5.2542	6.0506	4.1345	4.1421	3.9590
		P	$.0^{1}9486$	$.0^{1}8363$	$.0^{1}7508$	$.0^{1}9168$	$.0^{1}8511$	$.0^{1}8550$	$.0^{1}8611$	.1265	.1261	.1381
		log	$\bar{2}.9771$	$\bar{2}.9224$	$\bar{2}.8755$	$\bar{2}.9623$	$\bar{2}.9300$	$\bar{2}.9320$	$\bar{2}.9351$	$\bar{1}.1021$	$\bar{1}.1007$	$\bar{1}.1402$
9	2	S		4.7492	5.0398	4.6512	4.7115	5.0281	5.8010	4.1633	4.1675	4.1335
		P	.1067	$.0^{1}9305$	$.0^{1}8047$	$.0^{1}9772$	$.0^{1}9482$	$.0^{1}9543$	$.0^{1}9650$	.1247	.1245	.1266
		log	$\bar{1}.02816$	$\bar{2}.9687$	$\bar{2}.9056$	$\bar{2}.9900$	$\bar{2}.9769$	$\bar{2}.9797$	$\bar{2}.9845$	$\bar{1}.09587$	$\bar{1}.09517$	$\bar{1}.1024$

Note : 注
S = Statistic 統計量
P = Probability 確率
D F of Nass I = 2.1905
D F of Nass II = 2.6688

TABLE 6 TAIL PROBABILITIES IN 2×3 TABLES ACCUMULATED BY THE MAGNITUDE OF LR STATISTIC

表 6 尤度比統計量の大きさにより累積された2×3表の両側確率

x_{12}	x_{13}		Exact	$-2\log\lambda$	$-2K\log\lambda$	χ^2	Hald.	Nass I	Nass II	M/d	M/d'	χ^2_c
7	0	S		12.5880	11.6174	10.2361	10.2622	10.8372	12.2129	8.3059	8.5569	8.6297
		P	$.0^{2}4022$	$.0^{2}1847$	$.0^{2}3001$	$.0^{2}5988$	$.0^{2}5910$	$.0^{2}5546$	$.0^{2}4799$	$.0^{1}1572$	$.0^{1}1386$	$.0^{1}1337$
		log	$\bar{3}.6044$	$\bar{3}.2665$	$\bar{3}.4773$	$\bar{3}.7773$	$\bar{3}.7716$	$\bar{3}.7440$	$\bar{3}.6812$	$\bar{2}.1965$	$\bar{2}.1418$	$\bar{2}.1261$
8	0	S		12.1147	11.1805	9.8374	9.8589	10.4150	11.7470	7.9393	8.1793	8.2294
		P	$.0^{2}5007$	$.0^{2}2341$	$.0^{2}3734$	$.0^{2}7309$	$.0^{2}7231$	$.0^{2}6828$	$.0^{2}5990$	$.0^{1}1888$	$.0^{1}1675$	$.0^{1}1633$
		log	$\bar{3}.6996$	$\bar{3}.3694$	$\bar{3}.5722$	$\bar{3}.8639$	$\bar{3}.8592$	$\bar{3}.8343$	$\bar{3}.7774$	$\bar{2}.2760$	$\bar{2}.2240$	$\bar{2}.2130$
2	4	S		11.8223	10.9107	11.0145	11.0495	11.6613	13.1226	9.7251	9.7517	10.3133
		P	$.0^{2}5544$	$.0^{2}2709$	$.0^{2}4274$	$.0^{2}4057$	$.0^{2}3987$	$.0^{2}3696$	$.0^{2}3110$	$.0^{2}7731$	$.0^{2}7681$	$.0^{2}5761$
		log	$\bar{3}.7439$	$\bar{3}.4329$	$\bar{3}.6308$	$\bar{3}.6082$	$\bar{3}.6006$	$\bar{3}.5677$	$\bar{3}.4928$	$\bar{3}.8882$	$\bar{3}.8854$	$\bar{3}.7605$
9	5	S		11.1708	10.3094	10.5446	10.5743	11.1638	12.5735	8.9570	8.9766	9.7528
		P	$.0^{2}6332$	$.0^{2}3752$	$.0^{2}5772$	$.0^{2}5132$	$.0^{2}5056$	$.0^{2}4722$	$.0^{2}4041$	$.0^{1}1135$	$.0^{1}1124$	$.0^{2}7624$
		log	$\bar{3}.8016$	$\bar{3}.5743$	$\bar{3}.7613$	$\bar{3}.7103$	$\bar{3}.7038$	$\bar{3}.6741$	$\bar{3}.6065$	$\bar{2}.0550$	$\bar{2}.05077$	$\bar{3}.8822$
2	5	S		11.1708	10.3094	10.5446	10.5743	11.1638	12.5735	8.9570	8.9766	9.7528
		P	$.0^{2}7121$	$.0^{2}3752$	$.0^{2}5772$	$.0^{2}5132$	$.0^{2}5056$	$.0^{2}4722$	$.0^{2}4041$	$.0^{1}1135$	$.0^{1}1124$	$.0^{2}7624$
		log	$\bar{3}.8525$	$\bar{3}.5743$	$\bar{3}.7613$	$\bar{3}.7103$	$\bar{3}.7038$	$\bar{3}.6741$	$\bar{3}.6065$	$\bar{2}.0550$	$\bar{2}.05077$	$\bar{3}.8822$
10	1	S		11.0003	10.1521	9.7472	9.7676	10.3196	11.6416	8.5363	8.5631	8.7135
		P	$.0^{2}8067$	$.0^{2}4086$	$.0^{2}6245$	$.0^{2}7646$	$.0^{2}7568$	$.0^{2}7156$	$.0^{2}6298$	$.0^{1}1401$	$.0^{1}1382$	$.0^{1}1282$
		log	$\bar{3}.9067$	$\bar{3}.6113$	$\bar{3}.7955$	$\bar{3}.8834$	$\bar{3}.8790$	$\bar{3}.8547$	$\bar{3}.7992$	$\bar{2}.1464$	$\bar{2}.1405$	$\bar{2}.1079$
3	6	S		10.8397	10.0039	8.5843	8.5912	9.0884	10.2827	6.9776	7.1887	7.1764
		P	$.0^{2}9840$	$.0^{2}4428$	$.0^{2}6725$	$.0^{1}1368$	$.0^{1}1363$	$.0^{1}1311$	$.0^{1}1199$	$.0^{1}3054$	$.0^{1}2748$	$.0^{1}2765$
		log	$\bar{3}.9930$	$\bar{3}.6462$	$\bar{3}.8277$	$\bar{2}.1361$	$\bar{2}.1345$	$\bar{2}.1176$	$\bar{2}.07882$	$\bar{2}.4849$	$\bar{2}.4390$	$\bar{2}.4417$
7	6	S		10.5850	9.7688	8.3565	8.3608	8.8472	10.0165	6.8108	7.0168	6.9497
		P	$.0^{1}1181$	$.0^{2}5029$	$.0^{2}7564$	$.0^{1}1533$	$.0^{1}1529$	$.0^{1}1476$	$.0^{1}1360$	$.0^{1}3319$	$.0^{1}2994$	$.0^{1}3097$
		log	$\bar{2}.0723$	$\bar{3}.7015$	$\bar{3}.8788$	$\bar{2}.1855$	$\bar{2}.1844$	$\bar{2}.1691$	$\bar{2}.1335$	$\bar{2}.5210$	$\bar{2}.4763$	$\bar{2}.4909$
10	3	S		9.9859	9.2159	8.9118	8.9226	9.4351	10.6654	8.0776	8.0938	7.8410
		P	$.0^{1}1312$	$.0^{2}6786$	$.0^{2}9972$	$.0^{1}1161$	$.0^{1}1155$	$.0^{1}1106$	$.0^{1}1001$	$.0^{1}1762$	$.0^{1}1748$	$.0^{1}1983$
		log	$\bar{2}.1179$	$\bar{3}.8316$	$\bar{3}.9988$	$\bar{2}.0648$	$\bar{2}.0625$	$\bar{2}.04376$	$\bar{2}.00043$	$\bar{2}.2460$	$\bar{2}.2425$	$\bar{2}.2973$
10	2	S		9.3905	8.6664	8.3375	8.3416	8.8271	9.9943	7.5235	7.5395	7.2696
		P	$.0^{1}1490$	$.0^{2}9139$	$.0^{1}1313$	$.0^{1}1547$	$.0^{1}1544$	$.0^{1}1490$	$.0^{1}1374$	$.0^{1}2324$	$.0^{1}2306$	$.0^{1}2639$
		log	$\bar{2}.1732$	$\bar{3}.9609$	$\bar{2}.1183$	$\bar{2}.1895$	$\bar{2}.1886$	$\bar{2}.1732$	$\bar{2}.1380$	$\bar{2}.3662$	$\bar{2}.3629$	$\bar{2}.4214$
5	1	S		9.3266	8.6074	8.6080	8.6152	9.1135	10.3104	7.4971	7.5129	7.6196
		P	$.0^{1}1670$	$.0^{2}9435$	$.0^{1}1352$	$.0^{1}1351$	$.0^{1}1347$	$.0^{1}1295$	$.0^{1}1183$	$.0^{1}2355$	$.0^{1}2337$	$.0^{1}2215$
		log	$\bar{2}.2227$	$\bar{3}.9747$	$\bar{2}.1310$	$\bar{2}.1307$	$\bar{2}.1294$	$\bar{2}.1123$	$\bar{2}.07298$	$\bar{2}.3720$	$\bar{2}.3687$	$\bar{2}.3454$
4	6	S		8.6885	8.0185	6.4769	6.4593	6.8573	7.8200	5.3686	5.5312	5.0796
		P	$.0^{1}2143$	$.0^{1}1298$	$.0^{1}1815$	$.0^{1}3922$	$.0^{1}3957$	$.0^{1}3913$	$.0^{1}3806$	$.0^{1}6827$	$.0^{1}6294$	$.0^{1}7888$
		log	$\bar{2}.3310$	$\bar{2}.1133$	$\bar{2}.2589$	$\bar{2}.5935$	$\bar{2}.5974$	$\bar{2}.5925$	$\bar{2}.5805$	$\bar{2}.8342$	$\bar{2}.7989$	$\bar{2}.8970$
3	3	S		8.6828	8.0133	8.1144	8.1159	8.5909	9.7336	7.2820	7.2892	7.2712
		P	$.0^{1}2358$	$.0^{1}1302$	$.0^{1}1819$	$.0^{1}1730$	$.0^{1}1728$	$.0^{1}1674$	$.0^{1}1554$	$.0^{1}2623$	$.0^{1}2613$	$.0^{1}2637$
		log	$\bar{2}.3725$	$\bar{2}.1146$	$\bar{2}.2598$	$\bar{2}.2380$	$\bar{2}.2375$	$\bar{2}.2238$	$\bar{2}.1915$	$\bar{2}.4188$	$\bar{2}.4171$	$\bar{2}.4211$
6	6	S		8.5725	7.9115	6.3630	6.3441	6.7367	7.6868	5.2896	5.4498	4.9663
		P	$.0^{1}2855$	$.0^{1}1376$	$.0^{1}1914$	$.0^{1}4152$	$.0^{1}4192$	$.0^{1}4150$	$.0^{1}4049$	$.0^{1}7102$	$.0^{1}6555$	$.0^{1}8348$
		log	$\bar{2}.4556$	$\bar{2}.1386$	$\bar{2}.2819$	$\bar{2}.6183$	$\bar{2}.6224$	$\bar{2}.6180$	$\bar{2}.6073$	$\bar{2}.8514$	$\bar{2}.8166$	$\bar{2}.9216$
5	6	S		7.9509	7.3378	5.7365	5.7102	6.0734	6.9547	4.8133	4.9592	4.3429
		P	$.0^{1}3517$	$.0^{1}1877$	$.0^{1}2550$	$.0^{1}5680$	$.0^{1}5755$	$.0^{1}5738$	$.0^{1}5684$	$.0^{1}9012$	$.0^{1}8378$	.1140
		log	$\bar{2}.5462$	$\bar{2}.2735$	$\bar{2}.4065$	$\bar{2}.7543$	$\bar{2}.7600$	$\bar{2}.7588$	$\bar{2}.7547$	$\bar{2}.9548$	$\bar{2}.9231$	$\bar{1}.0569$
4	2	S		7.8327	7.2288	7.3123	7.3044	7.7417	8.7962	6.5362	6.5431	6.6468
		P	$.0^{1}3839$	$.0^{1}1991$	$.0^{1}2693$	$.0^{1}2583$	$.0^{1}2593$	$.0^{1}2538$	$.0^{1}2413$	$.0^{1}3808$	$.0^{1}3795$	$.0^{1}3603$
		log	$\bar{2}.5842$	$\bar{2}.2991$	$\bar{2}.4302$	$\bar{2}.4121$	$\bar{2}.4138$	$\bar{2}.4045$	$\bar{2}.3826$	$\bar{2}.5807$	$\bar{2}.5792$	$\bar{2}.5567$
9	1	S		7.2713	6.7106	6.7854	6.7714	7.1839	8.1805	5.7680	5.7800	5.7893
		P	$.0^{1}4312$	$.0^{1}2637$	$.0^{1}3490$	$.0^{1}3362$	$.0^{1}3385$	$.0^{1}3335$	$.0^{1}3218$	$.0^{1}5591$	$.0^{1}5558$	$.0^{1}5532$
		log	$\bar{2}.6347$	$\bar{2}.4211$	$\bar{2}.5428$	$\bar{2}.5266$	$\bar{2}.5296$	$\bar{2}.5231$	$\bar{2}.5076$	$\bar{2}.7475$	$\bar{2}.7449$	$\bar{2}.7429$
9	4	S		6.9364	6.4015	6.6288	6.6130	7.0181	7.9975	5.7618	5.7679	6.1155
		P	$.0^{1}4804$	$.0^{1}3117$	$.0^{1}4073$	$.0^{1}3362$	$.0^{1}3665$	$.0^{1}3617$	$.0^{1}3504$	$.0^{1}5609$	$.0^{1}5591$	$.0^{1}4699$
		log	$\bar{2}.6816$	$\bar{2}.4937$	$\bar{2}.6099$	$\bar{2}.5606$	$\bar{2}.5641$	$\bar{2}.5583$	$\bar{2}.5446$	$\bar{2}.7489$	$\bar{2}.7475$	$\bar{2}.6720$

TABLE 6 (Continued)　続き

X_{12}	X_{13}		Exact	$-2\log\lambda$	$-2K\log\lambda$	χ^2	Hald.	Nass I	Nass II	M/d	M/d'	χ^2_c
3	5	S		6.7079	6.1907	6.4437	6.4257	6.8221	7.7811	5.3844	5.3946	5.6727
		P	$.0^{1}5396$	$.0^{1}3495$	$.0^{1}4526$	$.0^{1}3988$	$.0^{1}4024$	$.0^{1}3981$	$.0^{1}3876$	$.0^{1}6773$	$.0^{1}6739$	$.0^{1}5864$
		log	$\bar{2}.7321$	$\bar{2}.5434$	$\bar{2}.6557$	$\bar{2}.6008$	$\bar{2}.6047$	$\bar{2}.6000$	$\bar{2}.5884$	$\bar{2}.8308$	$\bar{2}.8286$	$\bar{2}.7682$
3	5	S		6.7079	6.1907	6.4437	6.4257	6.8221	7.7811	5.3844	5.3946	5.6727
		P	$.0^{1}5987$	$.0^{1}3495$	$.0^{1}4526$	$.0^{1}3988$	$.0^{1}4024$	$.0^{1}3981$	$.0^{1}3876$	$.0^{1}6773$	$.0^{1}6739$	$.0^{1}5864$
		log	$\bar{2}.7772$	$\bar{2}.5434$	$\bar{2}.6557$	$\bar{2}.6008$	$\bar{2}.6047$	$\bar{2}.6000$	$\bar{2}.5884$	$\bar{2}.8308$	$\bar{2}.8286$	$\bar{2}.7682$
3	4	S		6.4795	5.9799	6.2871	6.2672	6.6563	7.5981	5.4434	5.4484	6.0415
		P	$.0^{1}6578$	$.0^{1}3917$	$.0^{1}5029$	$.0^{1}4313$	$.0^{1}4356$	$.0^{1}4317$	$.0^{1}4220$	$.0^{1}6576$	$.0^{1}6560$	$.0^{1}4877$
		log	$\bar{2}.8181$	$\bar{2}.5930$	$\bar{2}.7015$	$\bar{2}.6348$	$\bar{2}.6391$	$\bar{2}.6352$	$\bar{2}.6253$	$\bar{2}.8180$	$\bar{2}.8169$	$\bar{2}.6882$
6	1	S		6.4437	5.9468	6.1020	6.0800	6.4603	7.3818	5.1827	5.1924	5.1029
		P	$.0^{1}7240$	$.0^{1}3988$	$.0^{1}5113$	$.0^{1}4731$	$.0^{1}4784$	$.0^{1}4750$	$.0^{1}4665$	$.0^{1}7492$	$.0^{1}7455$	$.0^{1}7797$
		log	$\bar{2}.8597$	$\bar{2}.6008$	$\bar{2}.7087$	$\bar{2}.6750$	$\bar{2}.6798$	$\bar{2}.6767$	$\bar{2}.6689$	$\bar{2}.8746$	$\bar{2}.8724$	$\bar{2}.8919$
8	1	S		5.4330	5.0141	5.1907	5.1580	5.4955	6.3168	4.3219	4.3300	4.1878
		P	$.0^{1}8304$	$.0^{1}6611$	$.0^{1}8151$	$.0^{1}7462$	$.0^{1}7585$	$.0^{1}7603$	$.0^{1}7623$	.1152	.1148	.1232
		log	$\bar{2}.9193$	$\bar{2}.8203$	$\bar{2}.9112$	$\bar{2}.8729$	$\bar{2}.8800$	$\bar{2}.8810$	$\bar{2}.8821$	$\bar{1}.06145$	$\bar{1}.05994$	$\bar{1}.09061$
7	1	S		5.1783	4.7790	4.9628	4.9276	5.2542	6.0506	4.1345	4.1421	3.9590
		P	$.0^{1}9486$	$.0^{1}7508$	$.0^{1}9168$	$.0^{1}8363$	$.0^{1}8511$	$.0^{1}8550$	$.0^{1}8611$	.1265	.1261	.1381
		log	$\bar{2}.9771$	$\bar{2}.8755$	$\bar{2}.9623$	$\bar{2}.9224$	$\bar{2}.9300$	$\bar{2}.9320$	$\bar{2}.9351$	$\bar{1}.1021$	$\bar{1}.1007$	$\bar{1}.1402$
9	2	S		5.0398	4.6512	4.7492	4.7115	5.0281	5.8010	4.1633	4.1675	4.1335
		P	.1067	$.0^{1}8047$	$.0^{1}9772$	$.0^{1}9305$	$.0^{1}9482$	$.0^{1}9543$	$.0^{1}9650$	.1247	.1245	.1266
		log	$\bar{1}.0283$	$\bar{2}.9056$	$\bar{2}.9900$	$\bar{2}.9687$	$\bar{2}.9769$	$\bar{2}.9797$	$\bar{2}.9845$	$\bar{1}.09587$	$\bar{1}.09517$	$\bar{1}.1021$

Note : 注　S = Statistic 統計量
P = Probability 確率